固定界面模态综合法

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固定界面模态综合法

- 子结构模态分析
- 综合子结构
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子结构模态分析

子结构s的运动方程

$$\begin{bmatrix} \boldsymbol{M}_{ii}^{(s)} & \boldsymbol{M}_{ib}^{(s)} \\ \boldsymbol{M}_{bi}^{(s)} & \boldsymbol{M}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{a}}_i^{(s)} \\ \ddot{\boldsymbol{a}}_b^{(s)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ii}^{(s)} & \boldsymbol{K}_{ib}^{(s)} \\ \boldsymbol{K}_{bi}^{(s)} & \boldsymbol{K}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{Q}_i^{(s)} \\ \boldsymbol{Q}_b^{(s)} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(s)} \end{Bmatrix}$$

子结构s的自由振动方程

$$\begin{bmatrix} \boldsymbol{M}_{ii}^{(s)} & \boldsymbol{M}_{ib}^{(s)} \\ \boldsymbol{M}_{bi}^{(s)} & \boldsymbol{M}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{a}}_i^{(s)} \\ \ddot{\boldsymbol{a}}_b^{(s)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ii}^{(s)} & \boldsymbol{K}_{ib}^{(s)} \\ \boldsymbol{K}_{bi}^{(s)} & \boldsymbol{K}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(s)} \end{Bmatrix}$$

$$\boldsymbol{a}_i^{(s)} = \boldsymbol{K}_{ii}^{(s)-1} \boldsymbol{Q}_i^{(s)} - \boldsymbol{K}_{ii}^{(s)-1} \boldsymbol{K}_{ib}^{(s)} \boldsymbol{a}_b^{(s)}$$

可用子结构模态(界面固定)展开

子结构模态分析 — 固定界面模态

$$\begin{bmatrix} \boldsymbol{M}_{ii}^{(s)} & \boldsymbol{M}_{ib}^{(s)} \\ \boldsymbol{M}_{bi}^{(s)} & \boldsymbol{M}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{a}}_i^{(s)} \\ \ddot{\boldsymbol{a}}_b^{(s)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ii}^{(s)} & \boldsymbol{K}_{ib}^{(s)} \\ \boldsymbol{K}_{bi}^{(s)} & \boldsymbol{K}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(s)} \end{Bmatrix}$$

界面固定：$\boldsymbol{a}_b^{(s)} = \ddot{\boldsymbol{a}}_b^{(s)} = 0$

$$\boldsymbol{M}_{ii}^{(s)} \ddot{\boldsymbol{a}}_i^{(s)} + \boldsymbol{K}_{ii}^{(s)} \boldsymbol{a}_i^{(s)} = 0 \Longrightarrow \boldsymbol{\Lambda}^{(s)} \quad \boldsymbol{\Phi}_N^{(s)} \qquad (n_i^{(s)}) \text{ 个}$$

$$\boldsymbol{\Phi}_N^{(s)T} \boldsymbol{M}_{ii}^{(s)} \boldsymbol{\Phi}_N^{(s)} = \boldsymbol{I}_{n_i^{(s)}}$$

$$\boldsymbol{\Phi}_N^{(s)T} \boldsymbol{K}_{ii}^{(s)} \boldsymbol{\Phi}_N^{(s)} = \boldsymbol{\Lambda}^{(s)}$$

模态坐标向量

$$\boldsymbol{a}_i^{(s)} = \boldsymbol{\Phi}_N^{(s)} \boldsymbol{x}^{(s)} - (\boldsymbol{K}_{ii}^{(s)})^{-1} \boldsymbol{K}_{ib}^{(s)} \boldsymbol{a}_b^{(s)}$$

$$\begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_N^{(s)} & \boldsymbol{\Phi}_b^{(s)} \\ 0 & \boldsymbol{I}_{n_b^{(s)}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix}$$

$$\begin{aligned} \boldsymbol{\Phi}_b^{(s)} &= -(\boldsymbol{K}_{ii}^{(s)})^{-1} \boldsymbol{K}_{ib}^{(s)} \\ &= -\boldsymbol{K}_{ii}^{(s)-1} \boldsymbol{K}_{ib}^{(s)} \boldsymbol{I}_{n_b^{(s)}} \end{aligned}$$

约束模态： 第j列是在界面完全固定的情况下仅释放第j个界面自由度，并令它取单位值所得到的静态位移。

子结构模态分析 — 模态变换

$$\begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_N^{(s)} & \boldsymbol{\Phi}_b^{(s)} \\ 0 & \boldsymbol{I}_{n_b^{(s)}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix}$$

保留$k^{(s)}$个低阶主模态

$$\begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_k^{(s)} & \boldsymbol{\Phi}_b^{(s)} \\ 0 & \boldsymbol{I}_{n_b^{(s)}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_k^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \boldsymbol{T} \begin{Bmatrix} \boldsymbol{x}_k^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix}$$

$$\begin{bmatrix} \boldsymbol{M}_{ii}^{(s)} & \boldsymbol{M}_{ib}^{(s)} \\ \boldsymbol{M}_{bi}^{(s)} & \boldsymbol{M}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{a}}_i^{(s)} \\ \ddot{\boldsymbol{a}}_b^{(s)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ii}^{(s)} & \boldsymbol{K}_{ib}^{(s)} \\ \boldsymbol{K}_{bi}^{(s)} & \boldsymbol{K}_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{a}_i^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(s)} \end{Bmatrix}$$

$$\bar{\boldsymbol{M}}^{(s)} \begin{Bmatrix} \ddot{\boldsymbol{x}}_k^{(s)} \\ \ddot{\boldsymbol{a}}_b^{(s)} \end{Bmatrix} + \bar{\boldsymbol{K}}^{(s)} \begin{Bmatrix} \boldsymbol{x}_K^{(s)} \\ \boldsymbol{a}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(s)} \end{Bmatrix} \qquad \begin{aligned} \bar{\boldsymbol{M}}^{(s)} &= \boldsymbol{T}^T \boldsymbol{M}^{(s)} \boldsymbol{T} \\ \bar{\boldsymbol{K}}^{(s)} &= \boldsymbol{T}^T \boldsymbol{K}^{(s)} \boldsymbol{T} \end{aligned}$$

子结构模态分析 — 模态变换

$$\bar{\boldsymbol{M}}^{(s)} = \begin{bmatrix} \bar{\boldsymbol{M}}_{kk}^{(s)} & \bar{\boldsymbol{M}}_{kb}^{(s)} \\ \bar{\boldsymbol{M}}_{bk}^{(s)} & \bar{\boldsymbol{M}}_{bb}^{(s)} \end{bmatrix}$$

$$\begin{aligned} \bar{\boldsymbol{M}}_{kk}^{(s)} &= \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{M}_{ii}^{(s)} \boldsymbol{\Phi}_k^{(s)} = \boldsymbol{I}_k^{(s)} \\ \bar{\boldsymbol{M}}_{bb}^{(s)} &= \boldsymbol{M}_{bb}^{(s)} + \boldsymbol{\Phi}_b^{(s)T} \boldsymbol{M}_{ib}^{(s)} + \boldsymbol{M}_{bi}^{(s)} \boldsymbol{\Phi}_b^{(s)} + \boldsymbol{\Phi}_b^{(s)T} \boldsymbol{M}_{ii} \boldsymbol{\Phi}_b^{(s)} \\ \bar{\boldsymbol{M}}_{kb}^{(s)} &= (\bar{\boldsymbol{M}}_{bk}^{(s)})^T = \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{M}_{ii}^{(s)} \boldsymbol{\Phi}_b^{(s)} + \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{M}_{ib}^{(s)} \end{aligned}$$

$$\bar{\boldsymbol{K}}^{(s)} = \begin{bmatrix} \bar{\boldsymbol{K}}_{kk}^{(s)} & \bar{\boldsymbol{K}}_{kb}^{(s)} \\ \bar{\boldsymbol{K}}_{bk}^{(s)} & \bar{\boldsymbol{K}}_{bb}^{(s)} \end{bmatrix}$$

$$\begin{aligned} \bar{\boldsymbol{K}}_{kk}^{(s)} &= \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{K}_{ii}^{(s)} \boldsymbol{\Phi}_k^{(s)} = \boldsymbol{\Lambda}_k^{(s)} \\ \bar{\boldsymbol{K}}_{bb}^{(s)} &= \boldsymbol{K}_{bb}^{(s)} + \boldsymbol{\Phi}_b^{(s)T} \boldsymbol{K}_{ib}^{(s)} + \boldsymbol{K}_{bi}^{(s)} \boldsymbol{\Phi}_b^{(s)} + \boldsymbol{\Phi}_b^{(s)T} \boldsymbol{K}_{ii}^{(s)} \boldsymbol{\Phi}_b^{(s)} \\ \bar{\boldsymbol{K}}_{kb}^{(s)} &= \bar{\boldsymbol{K}}_{bk}^{(s)T} = \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{K}_{ii}^{(s)} \boldsymbol{\Phi}_b^{(s)} + \boldsymbol{\Phi}_k^{(s)T} \boldsymbol{K}_{ib}^{(s)} \\ &= \boldsymbol{\Phi}_k^{(s)T} [\boldsymbol{K}_{ii}^{(s)} \boldsymbol{\Phi}_b^{(s)} + \boldsymbol{K}_{ib}^{(s)}] \\ &= 0 \end{aligned}$$

综合子结构 — 模态综合

模态坐标空间内子结构的自由振动方程

$$\begin{bmatrix} \boldsymbol{I}_k^{(1)} & \bar{\boldsymbol{M}}_{kb}^{(1)} \\ \bar{\boldsymbol{M}}_{bk}^{(1)} & \bar{\boldsymbol{M}}_{bb}^{(1)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_k^{(1)} \\ \ddot{\boldsymbol{a}}_b^{(1)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{\Lambda}_k^{(1)} & 0 \\ 0 & \bar{\boldsymbol{K}}_{bb}^{(1)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_k^{(1)} \\ \boldsymbol{a}_b^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(1)} \end{Bmatrix} \quad k^{(1)} + n_b^{(1)}\text{个}$$

$$\begin{bmatrix} \boldsymbol{I}_k^{(2)} & \bar{\boldsymbol{M}}_{kb}^{(2)} \\ \bar{\boldsymbol{M}}_{bk}^{(2)} & \bar{\boldsymbol{M}}_{bb}^{(2)} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_k^{(2)} \\ \ddot{\boldsymbol{a}}_b^{(2)} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{\Lambda}_k^{(2)} & 0 \\ 0 & \bar{\boldsymbol{K}}_{bb}^{(2)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_k^{(2)} \\ \boldsymbol{a}_b^{(2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{R}_b^{(2)} \end{Bmatrix}$$

两个子结构的系统：$\boldsymbol{a}_b^{(1)} = \boldsymbol{a}_b^{(2)} = \boldsymbol{a}_b \qquad \boldsymbol{R}_b^{(1)} + \boldsymbol{R}_b^{(2)} = 0$

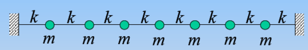
$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = 0$$

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{I}_k^{(1)} & 0 & \bar{\boldsymbol{M}}_{kb}^{(1)} \\ 0 & \boldsymbol{I}_k^{(2)} & \bar{\boldsymbol{M}}_{kb}^{(2)} \\ \bar{\boldsymbol{M}}_{bk}^{(1)} & \bar{\boldsymbol{M}}_{bk}^{(2)} & \bar{\boldsymbol{M}}_{bb}^{(1)} + \bar{\boldsymbol{M}}_{bb}^{(2)} \end{bmatrix} \quad \boldsymbol{K} = \begin{bmatrix} \boldsymbol{\Lambda}_k^{(1)} & 0 & 0 \\ 0 & \boldsymbol{\Lambda}_k^{(2)} & 0 \\ 0 & 0 & \bar{\boldsymbol{K}}_{bb}^{(1)} + \bar{\boldsymbol{K}}_{bb}^{(2)} \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_k^{(1)} \\ \boldsymbol{x}_k^{(2)} \\ \boldsymbol{a}_b \end{bmatrix}$$

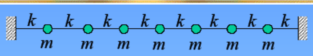
- 阶数远小于直接在物理坐标中建立的整体系统运动方程的阶数
- 类似地，可求解强迫振动问题

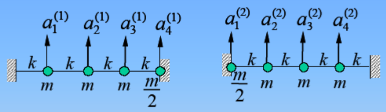
例4-1

长为L、质量为M、张力为T的弦用集中质量法等分为8段，在7个等分点上各有集中质量$m = M/8$，试用固定界面模态综合法求此弦的低阶自振频率。



例4-1





1号子结构 $\boldsymbol{a}_i^{(1)} = [a_1^{(1)} \quad a_2^{(1)} \quad a_3^{(1)}]^T \qquad \boldsymbol{a}_b^{(1)} = [a_4^{(1)}]^T$

$$\boldsymbol{K}_{ii}^{(1)} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad \boldsymbol{K}_{ib}^{(1)} = k\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad \boldsymbol{K}_{bb}^{(1)} = [1]$$

$$k = \frac{8T}{L}$$

$$\boldsymbol{M}_{ii}^{(1)} = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{M}_{ib}^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{M}_{bb}^{(1)} = \begin{bmatrix} \frac{1}{2} \end{bmatrix}$$

例4-1

取两阶模态参加模态综合 **取$k = 1$, $m = 1$**

$$\boldsymbol{\Phi}_k^{(1)} = \begin{bmatrix} 1/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & 0 \\ 1/2 & -\sqrt{2}/2 \end{bmatrix} \qquad \boldsymbol{x}_k^{(1)} = [x_1^{(1)} \quad x_2^{(1)}]^T$$

该子结构的约束模态为

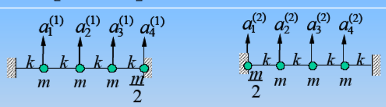
$$\boldsymbol{\Phi}_b^{(1)} = -\left(\boldsymbol{K}_{ii}^{(1)}\right)^{-1} \boldsymbol{K}_{ib}^{(s)} = \begin{bmatrix} 1/4 \\ 2/4 \\ 3/4 \end{bmatrix}$$

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{\Phi}_k^{(s)} & \boldsymbol{\Phi}_b^{(s)} \\ 0 & \boldsymbol{I}_{n_b^{(s)}} \end{bmatrix}$$

例4-1

$$\bar{\boldsymbol{M}}_{kk}^{(1)} = \boldsymbol{I}_k^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \bar{\boldsymbol{M}}_{bb}^{(1)} = [\tfrac{11}{8}] \qquad \bar{\boldsymbol{M}}_{kb}^{(1)} = \begin{bmatrix} (2+\sqrt{2})/4 \\ -\sqrt{2}/4 \end{bmatrix}$$

$$\bar{\boldsymbol{K}}_{kk}^{(1)} = \boldsymbol{\Lambda}_k^{(1)} = \begin{bmatrix} 2-\sqrt{2} & 0 \\ 0 & 2 \end{bmatrix}, \quad \bar{\boldsymbol{K}}_{bb}^{(1)} = [\tfrac{1}{4}], \quad \bar{\boldsymbol{K}}_{kb}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$\boldsymbol{a}_i^{(1)} = [a_1^{(1)} \quad a_2^{(1)} \quad a_3^{(1)}]^T$

$\boldsymbol{a}_b^{(1)} = [a_4^{(1)}]^T$

$\boldsymbol{a}_i^{(2)} = [a_4^{(2)}\ a_3^{(2)}\ a_2^{(2)}]^T$

$\boldsymbol{a}_b^{(2)} = [a_1^{(2)}]$

$$\boldsymbol{\Phi}_b^{(2)} = \boldsymbol{\Phi}_b^{(1)}, \quad \boldsymbol{\Phi}_k^{(2)} = \boldsymbol{\Phi}_k^{(1)}$$

$$\bar{\boldsymbol{M}}_{kk}^{(2)} = \bar{\boldsymbol{M}}_{kk}^{(1)}, \quad \bar{\boldsymbol{M}}_{bb}^{(2)} = \bar{\boldsymbol{M}}_{bb}^{(1)}, \quad \bar{\boldsymbol{M}}_{kb}^{(2)} = \bar{\boldsymbol{M}}_{kb}^{(1)}$$

$$\bar{\boldsymbol{K}}_{kk}^{(2)} = \bar{\boldsymbol{K}}_{kk}^{(1)}, \quad \bar{\boldsymbol{K}}_{bb}^{(2)} = \bar{\boldsymbol{K}}_{bb}^{(1)}, \quad \bar{\boldsymbol{K}}_{kb}^{(2)} = \bar{\boldsymbol{K}}_{kb}^{(1)}$$

例4-1

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = 0 \qquad \boldsymbol{x} = [x_1^{(1)}\ x_2^{(1)}\ x_1^{(2)}\ x_2^{(2)}\ a_b]^T$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{2+\sqrt{2}}{4} \\ 0 & 1 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 1 & 0 & \frac{2+\sqrt{2}}{4} \\ 0 & 0 & 0 & 1 & -\frac{\sqrt{2}}{4} \\ \frac{2+\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{2+\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{11}{4} \end{bmatrix}$$

例4-1

$$K=\begin{bmatrix} 2-\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2-\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$\omega_1=0.39028$ $\omega_2=0.76537$

$\omega_{1o}=0.39018$ $\omega_{2o}=0.76537$

讨论

- 固定界面模态综合法缺点
  - 保留了所有界面自由度
  - 难以与实验结合
- 解决办法
  - 减少子结构个数
  - 多重子结构法