Adaptive Finite Element Material Point Method

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Acknowledgement

Y Liu, YP Lian, S Ma, P Huang, ZT Ma, PF Yang, YT Zhang, WW Gong, HK Wang, XJ Wang, SZ Zhou, JG Li, P Liu, XX Cui, F Zhang, HY Zhao

Outline

- **Introduction**
- MPM3D
- FEM-MPM Coupling
  - Hybrid FE-MP method
  - Coupled FE-MP method
  - Material Point Finite Element Method
  - Adaptive FE-MP method
- Concluding Remarks

Introduction

- Intense dynamic loading
  - High velocity impact
  - Blast
  - Cutting
- Highly nonlinear
  - Geometrical nonlinearity
  - Material nonlinearity
  - Boundary condition nonlinearity
- Deformation is highly localized

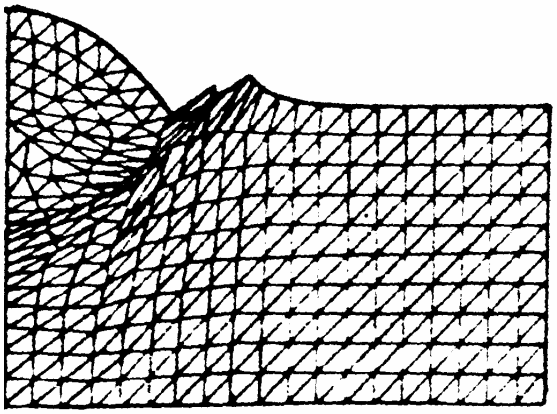


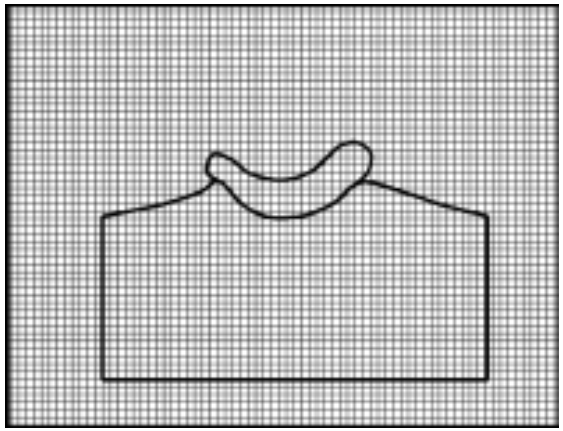


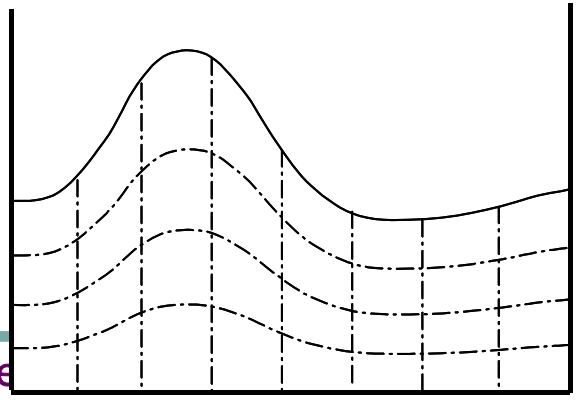


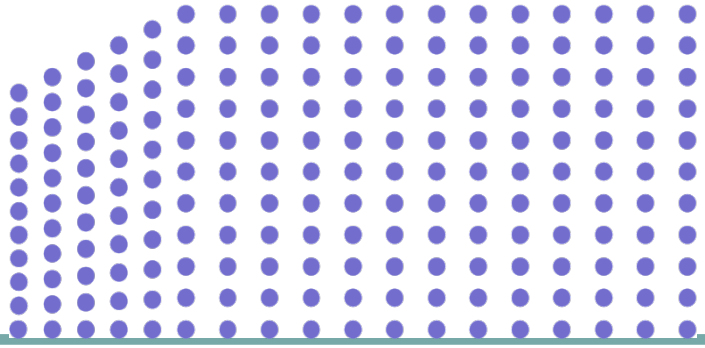
Introduction

- Numerical methods
  - Lagrangian method: Element distortion
  - Eulerian method: Material interface
  - Hybrid Lagrangian-Eulerian methods
    - Arbitrary Lagrangian Eulerian (ALE)
    - Particle In Cell (PIC)
  - Meshfree methods
    - SPH
    - EFG
    - RKPM
    - MLPG
    - … …



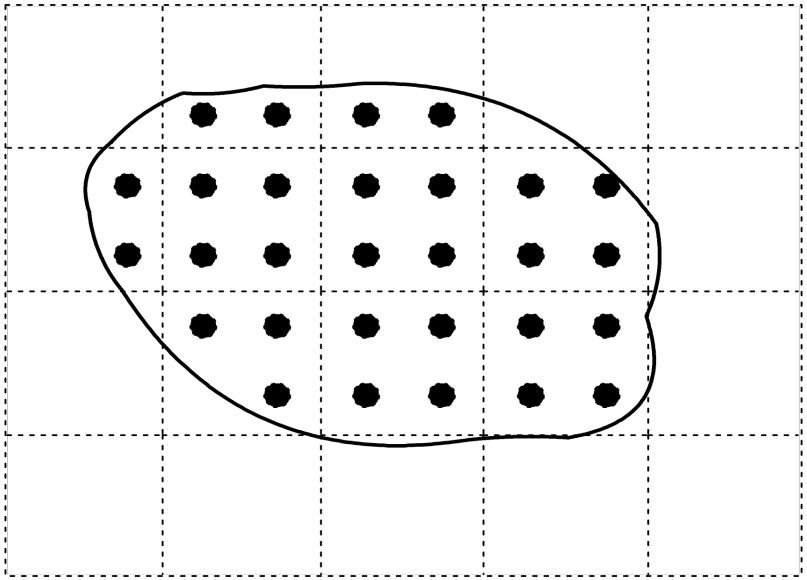






Introduction

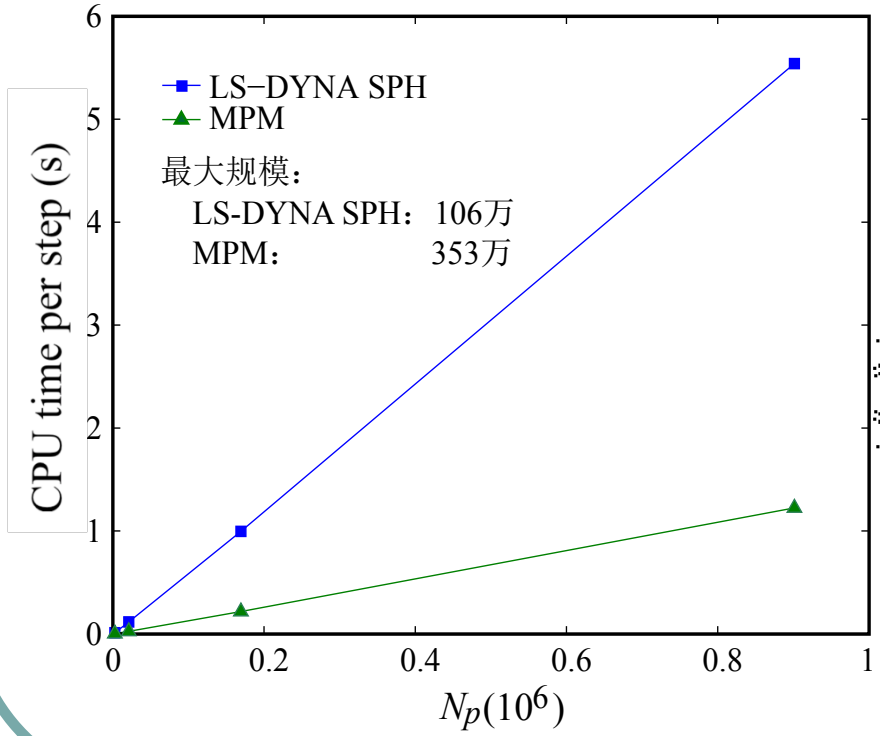
- Material Point Method (MPM, Sulsky, Chen et. al. 1994)
  - An extension of the FLIP PIC method to solid mechanics
  - Discretizes a material domain by particles moving through an Eulerian background mesh
  - Fully Lagrangian particle method
    - Particles carry all state variables
    - Grid carries no permanent information

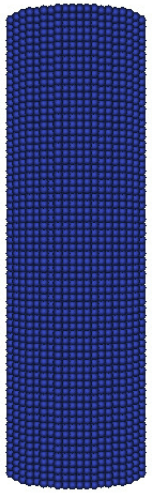


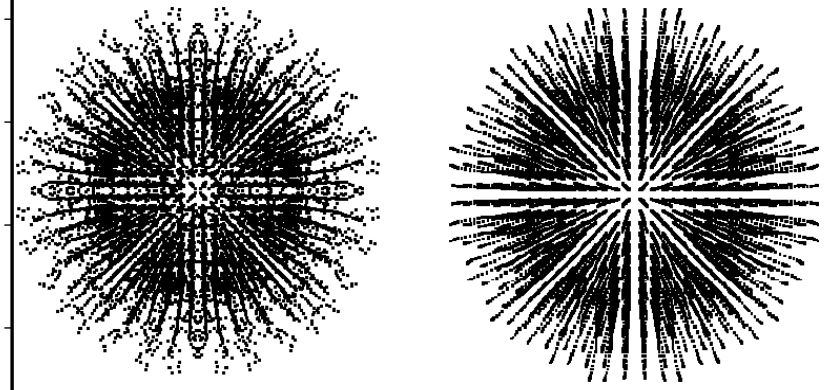
Introduction

- Much more efficient than SPH
  - The critical time step size depend on the cell size
  - No neighboring particles search

S Ma， X Zhang et al. Comparison study of MPM and SPH in modeling hypervelocity impact problems. ***International Journal of Impact Engineering*** 36: 272-282, 2009







SPH MPM

Final configuration of the bar (top view)

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  - Adaptive FE-MP method
- Concluding Remarks

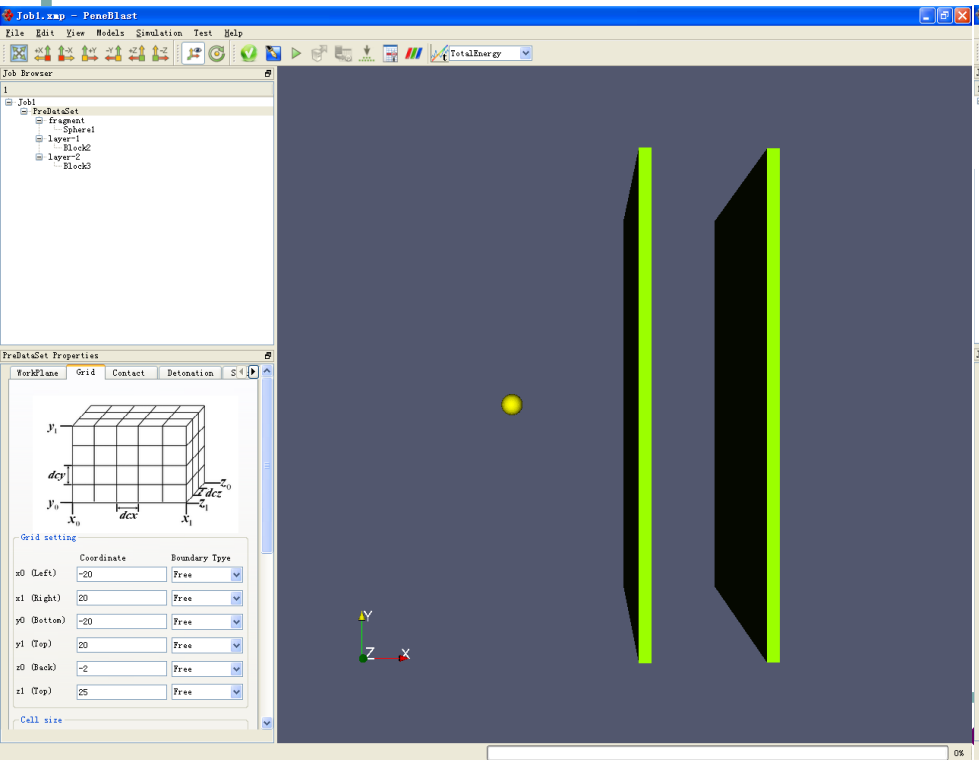
MPM3D

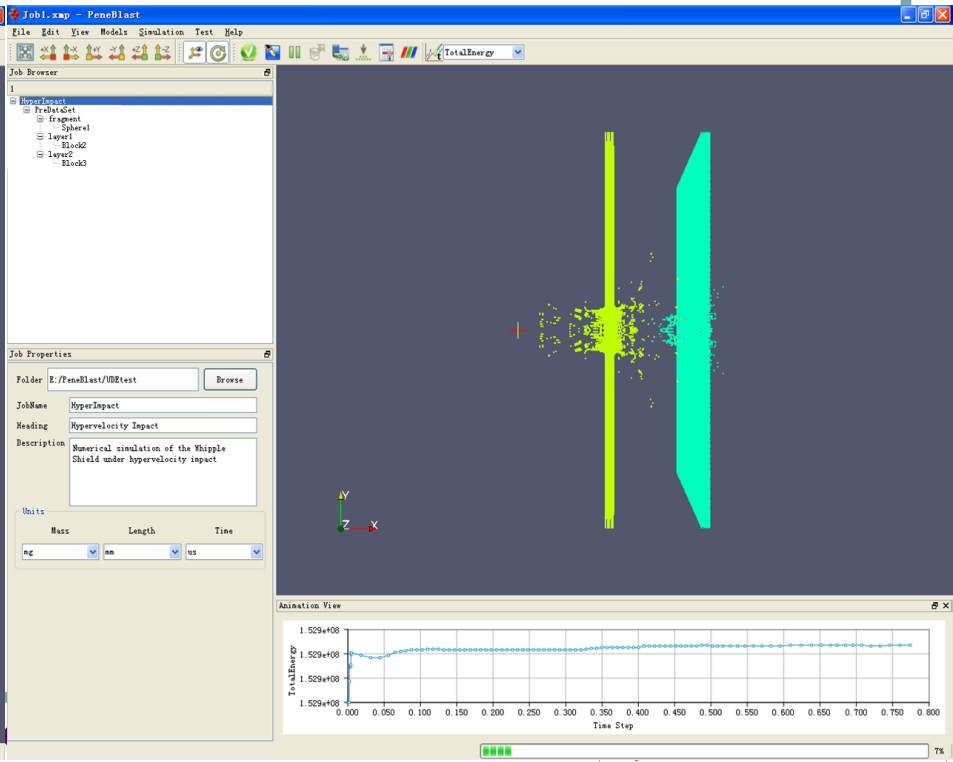
- MPM3D solver
  - Algorithm
    - MPM / GIMP (USF, USL & MUSL)
    - FEM
    - HFEMP、CFEMP and AFEMP
  - Material models
    - Elasticity, plasticity (hardening, temperature, strain rate)
    - Fluid, High explosive
    - Concrete, Ceramic, rubber, Foam, Soil …
  - Equation of states
    - Solid
    - Fluid
    - High explosive
  - Failure models
    - Hydrodynamic tensile failure; Effective plastic strain
    - Principal stress failure; Principal strain failure

MPM3D-F90: Open source
http://www.mpm3d.com

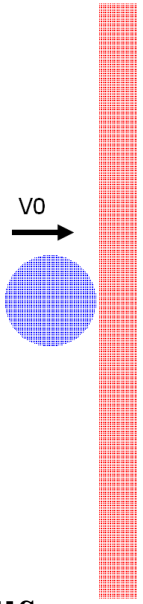
MPM3D

- Graphical User Interface (GUI)
  - Preprocessor
  - Solution process monitor
  - Postprocessor
    - Solver ➔ ParaView (http://www.paraview.org)
  - Cross platform (Windows, Linux, Mac OS X)



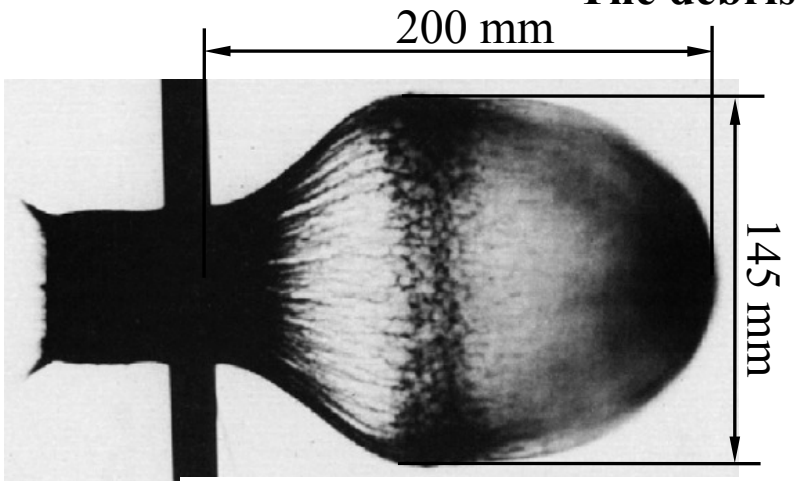


- Projectile: Lead sphere, r =7.5mm, m = 20g
- Target: Lead plate, t = 6.35 mm
- Impact velocity: v_0 = 6.58 km/s
- Material model
  - Elastic-plastic constitutive model
  - Mie-Grüneisen EOS
- Material failure model
  - Tensile Failure (P_{min} = −1500 MPa)
  - Plastic Strain Failure (ε_{pmax}= 3.0)

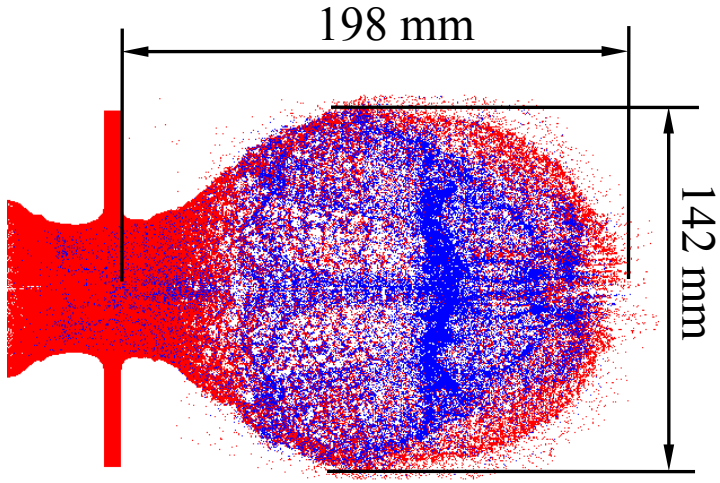


2 Quad-Core Xeon
2.66 GHz processors
(OpenMP): 1.93h

The debris cloud at 30.6 μs



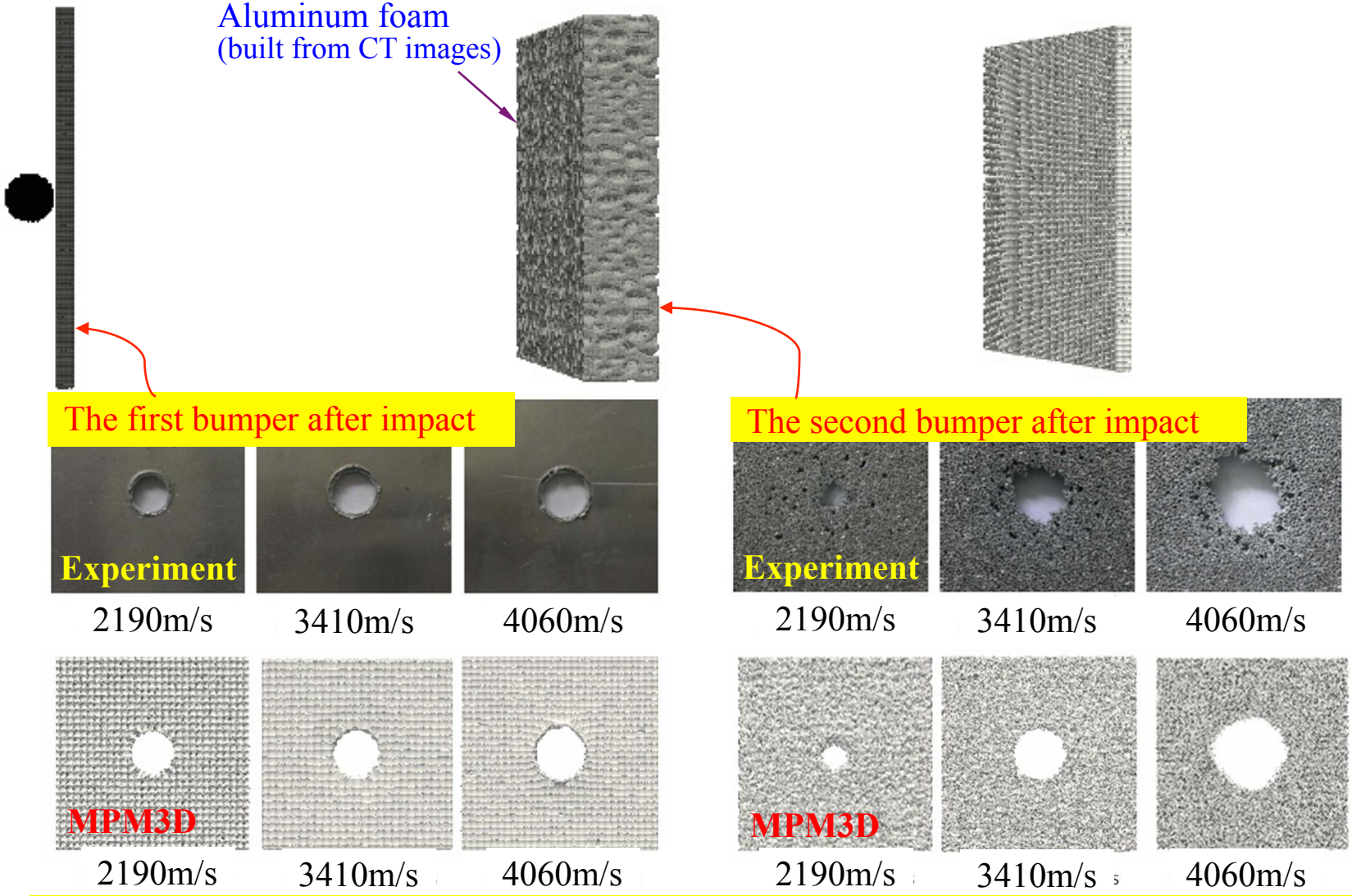
Experimental radiograph (Anderson, 1990)



13,542,030 particles

MPM3D

Application: Space debris shielding



WW Gong, Y Liu, X Zhang. Numerical Investigation on Dynamical Response of Aluminum Foam Subject to Hypervelocity Impact With Material Point Method. ***CMES*** 83(5): 527-545, 2012

MPM3D

Application: Penetration

Projectile: Steel
- Length: 88.9 mm
- Diamete: 12.9 mm
- Velocity: 575 m/s
- Particles: 13,314

Target: Aluminium
- Thickness: 26.3mm
- Inclination: 30°
- Particles: 187,550



Projectile's residual velocity
- Experiment: 455 m/s
- MPM3D: 453.4 m/s

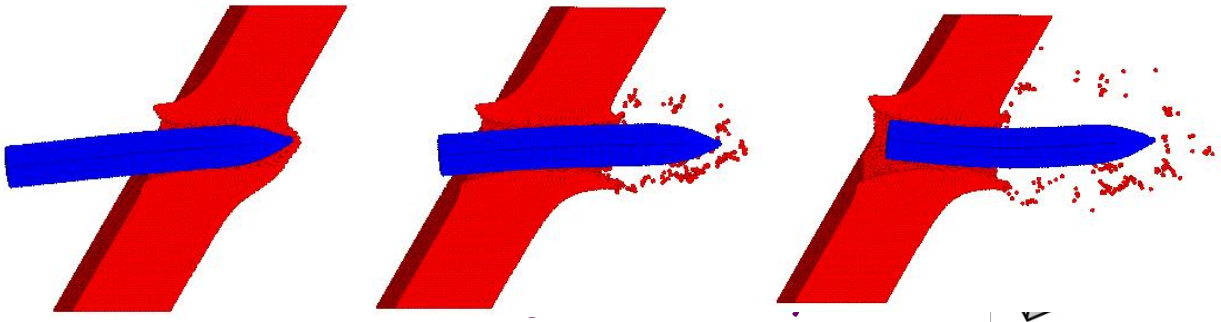
P Huang, X Zhang et al. Contact Algorithms for the Material Point Method in Impact and Penetration Simulation. ***IJNME*** 85(4): 498-517, 2011

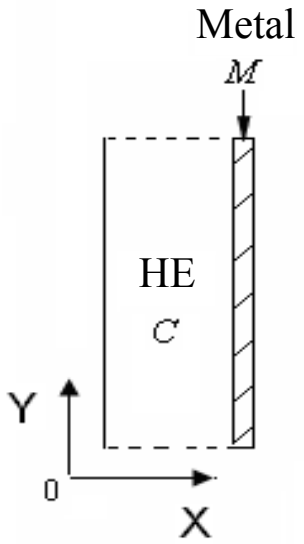
Experiment: Piekutowski, Forrestal et al. ***IJIE*** 1996; 18: 877-887

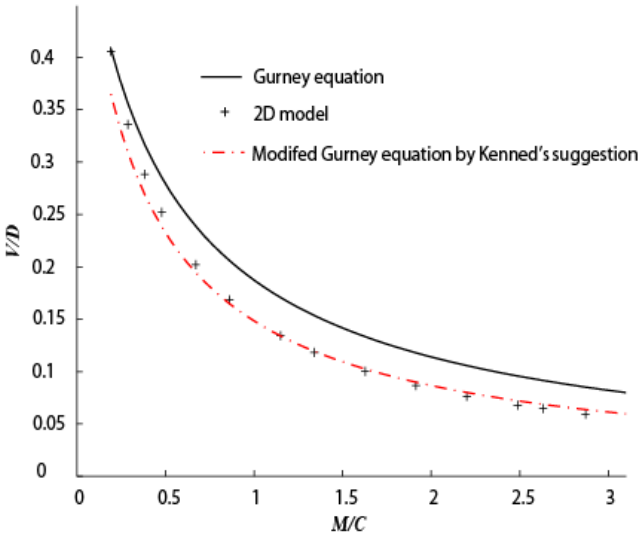


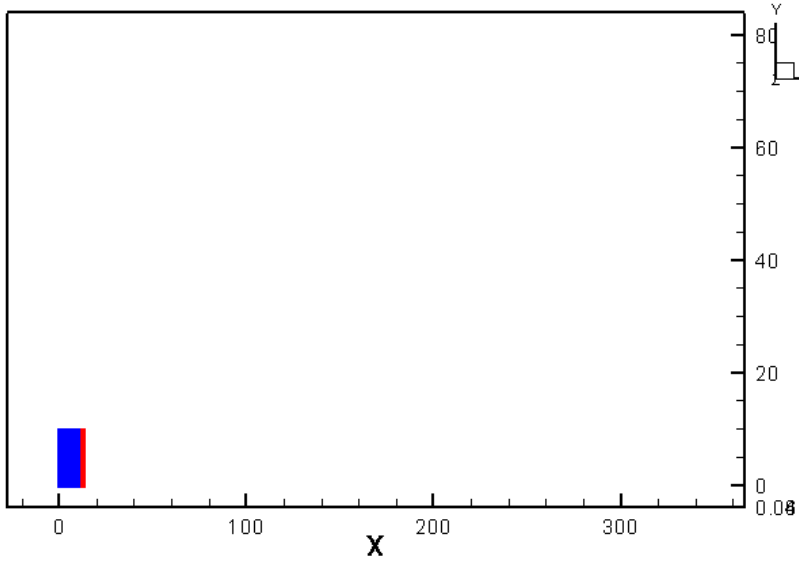
t = 82.9 μs t =152.8 μs t =194.4 μs

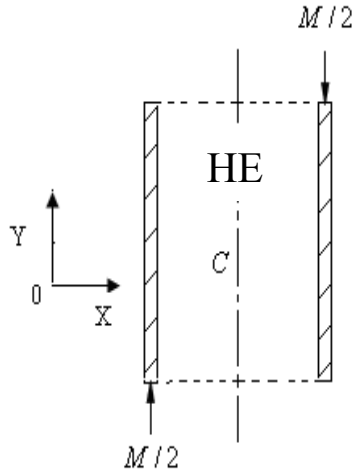
MPM3D

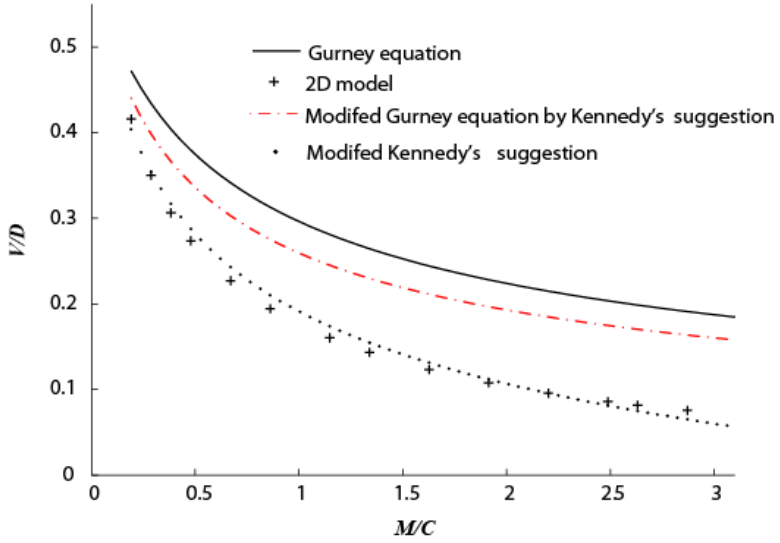










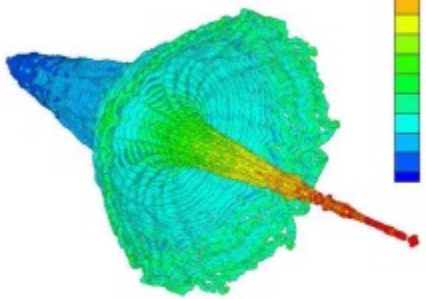


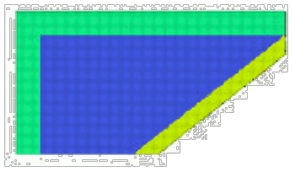
YP Lian, X Zhang et al. Numerical Simulation of Explosively Driven Metal by Material Point Method. ***International Journal of Impact Engineering***, 38(4): 238-246, 2011

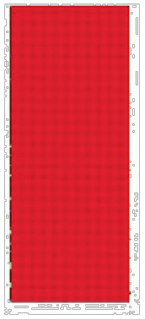
MPM3D

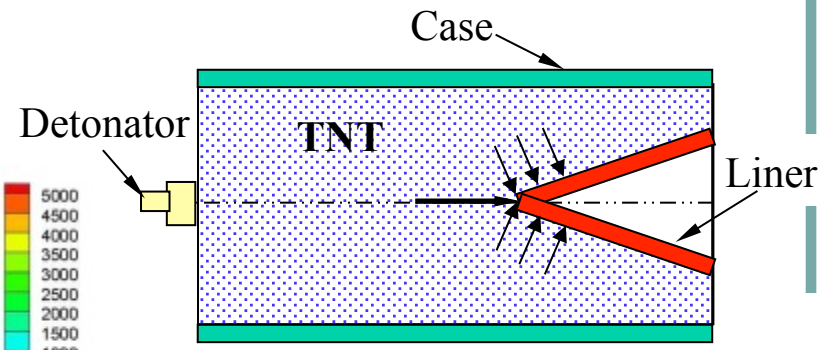
Application: Shaped charge

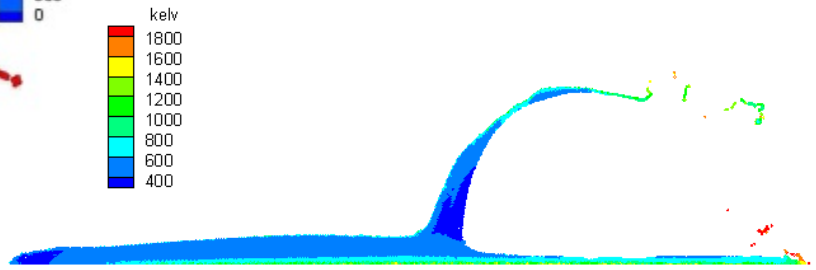
- Shaped charge
  - A cylinder of high explosive with a cone-shape thin metal liner in one end
  - Intense pressure ➔ Collapse of the liner ➔ A jet with extreme high velocity (> 10 km/s)
- Application
  - Armor penetration
  - Metal cutting
  - Oil well perforation



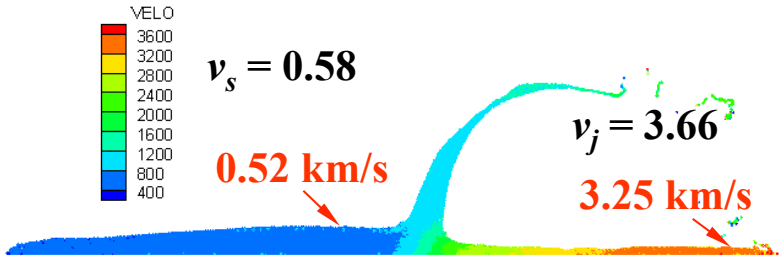








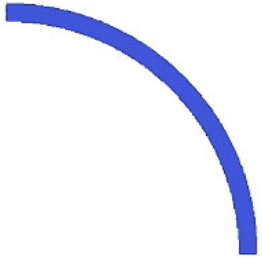
Temperature distribution



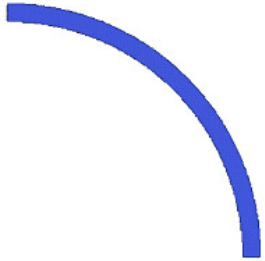
Velocity distribution

S Ma, X Zhang et al. Simulation of high explosive explosion using adaptive material point method, ***CMES: Computer Modeling in Engineering & Sciences***, 39(2): 101-123, 2009

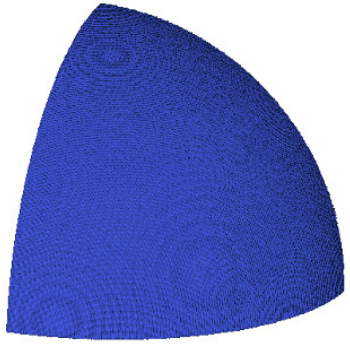
Application: Fragmentation driven by explosion



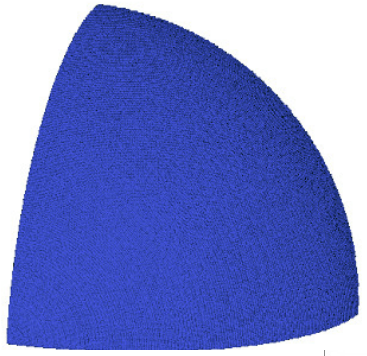
Weibull random failure



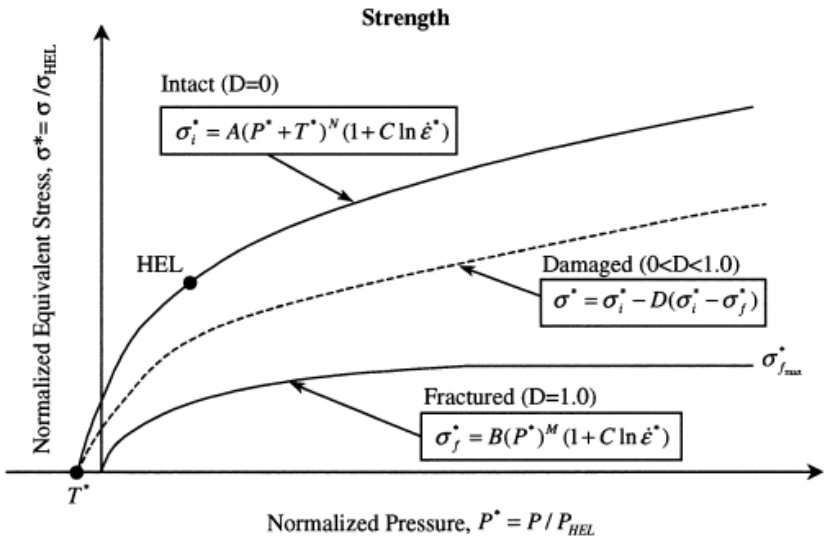
Gurson model

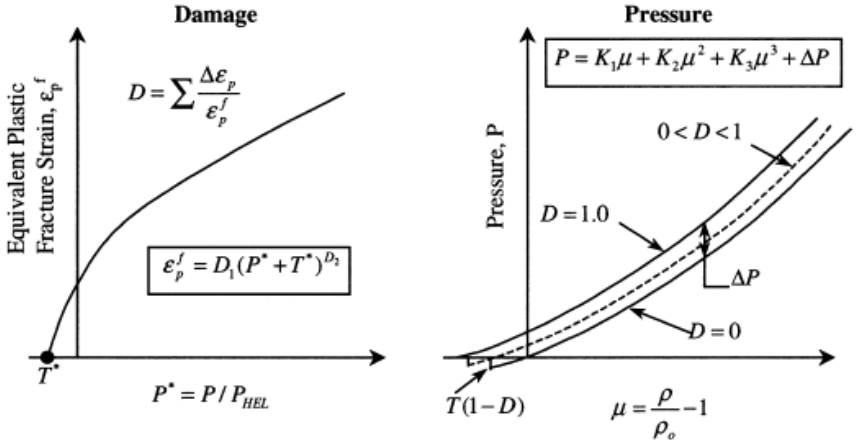


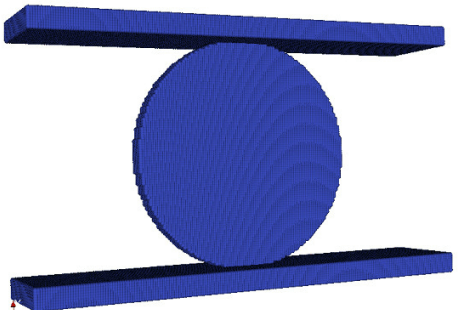
PF Yang, Y Liu, X Zhang et al. Simulation of Fragmentation with Material Point Method Based on Gurson Model and Random Failure. ***CMES: Computer Modeling in Engineering & Sciences***, 85(3): 207-236, 2012.



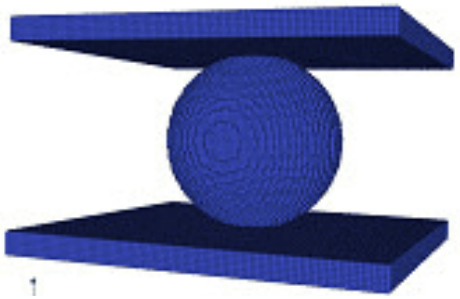
LS-DYNA MAT_110: JH-2







Crush of a ceramic plate

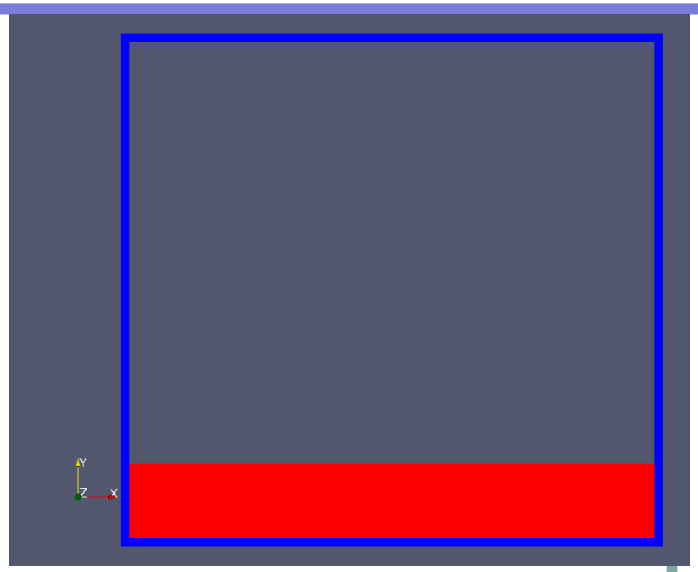


Crush of a ceramic ball

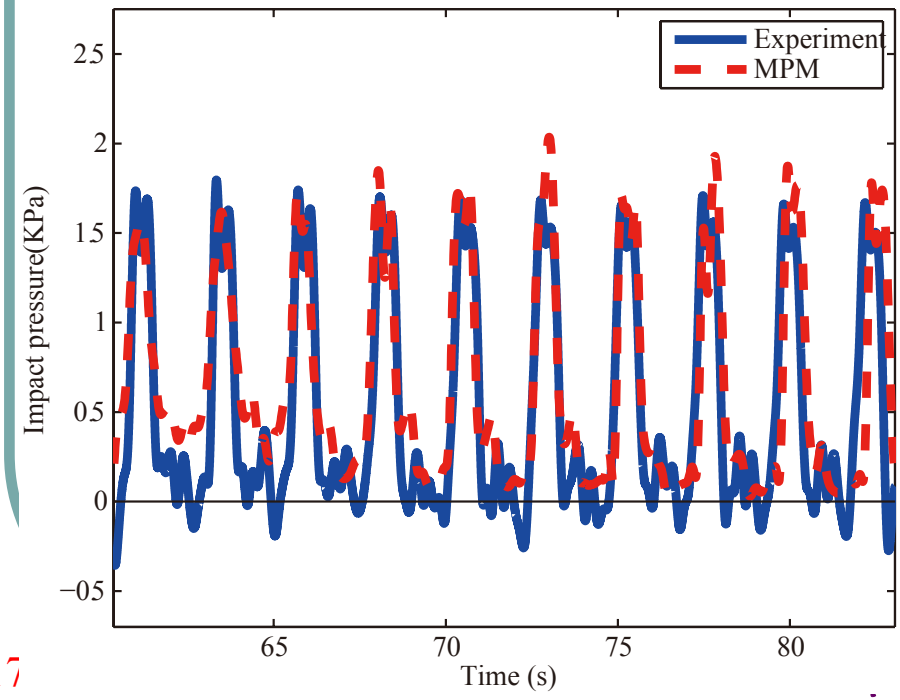


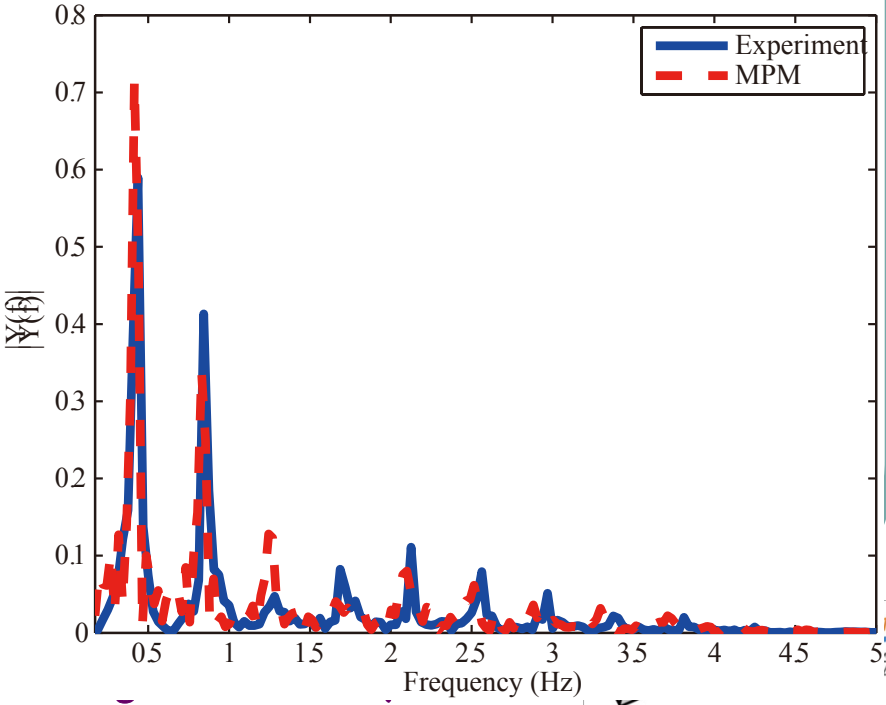
Cutting a ceramic block





Collaborated with Y Hamamoto. IHI, Japan.



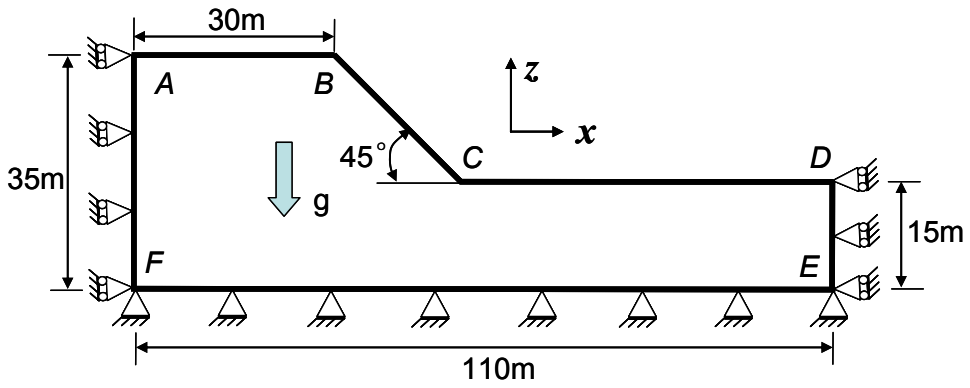


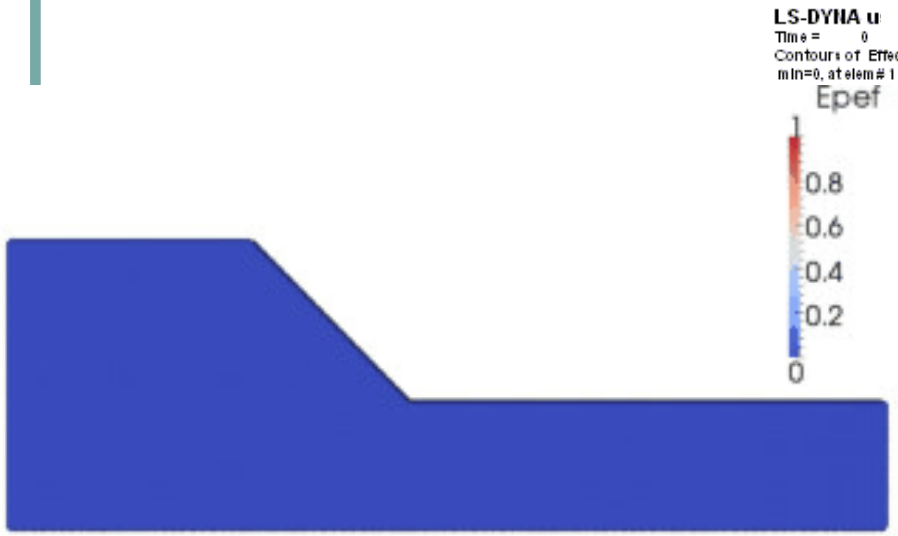
- **Non-cohesive soil**

  Drucker-Prager model

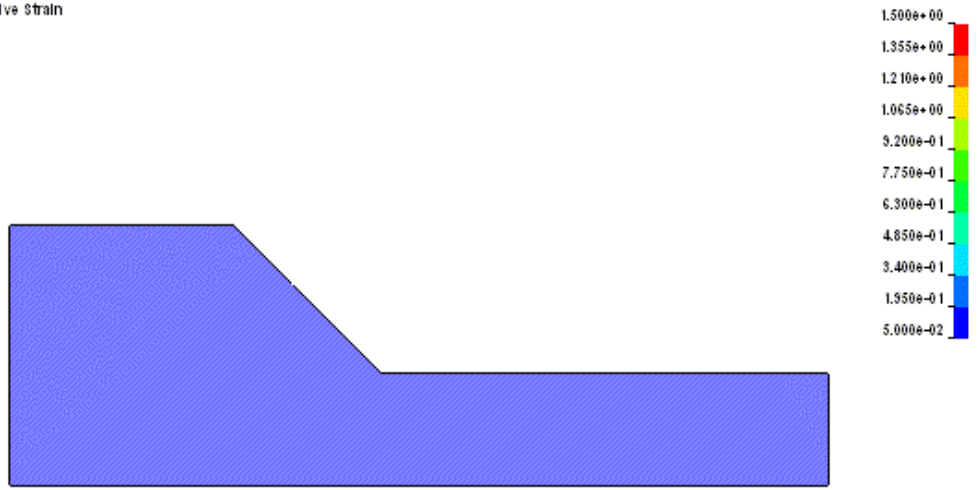
  $\phi = 20°$; $\psi = 0^0$; $c = 0.1$ kPa

  E=70MPa, ν =0.3, ρ =2.1 g/cm3



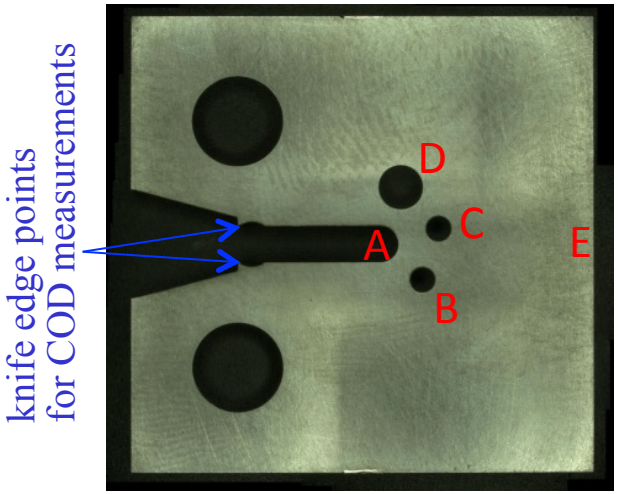


MPM3D： 9 minutes

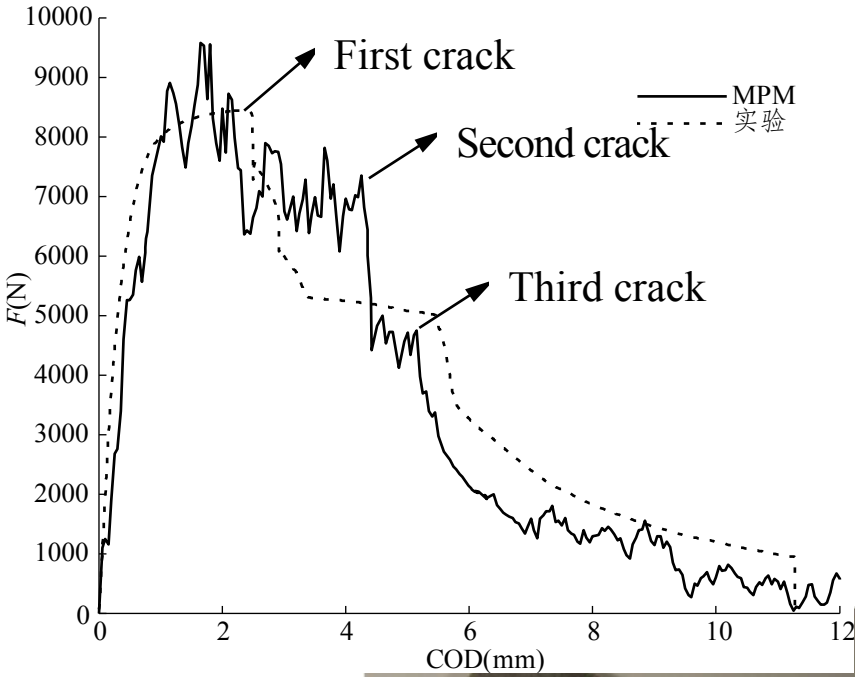


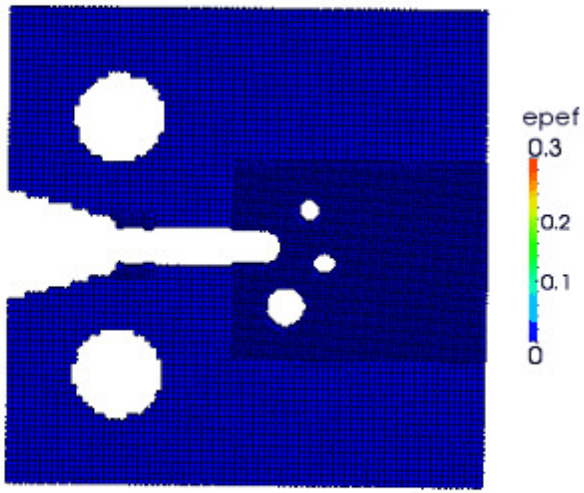
LS-DYNA (FEM): 468 minutes

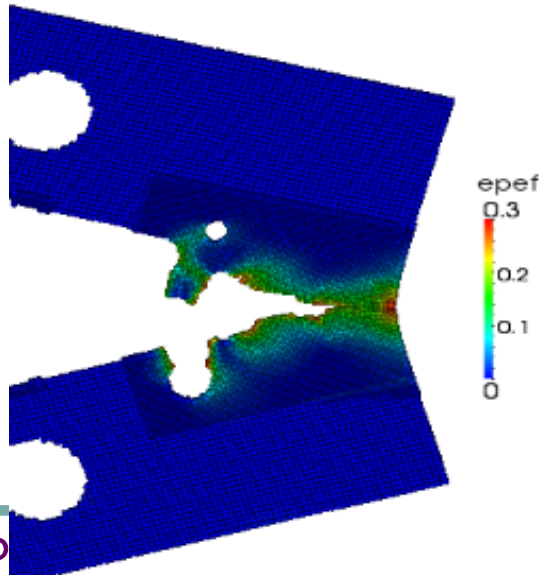
Fracture of ductile metal
The 2012 Sandia Fracture Challenge



Alloy 15-5 PH





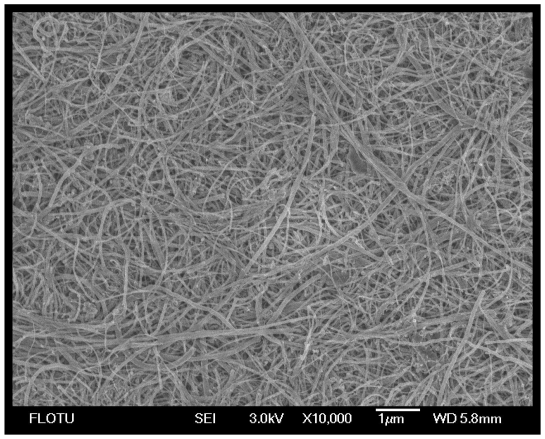


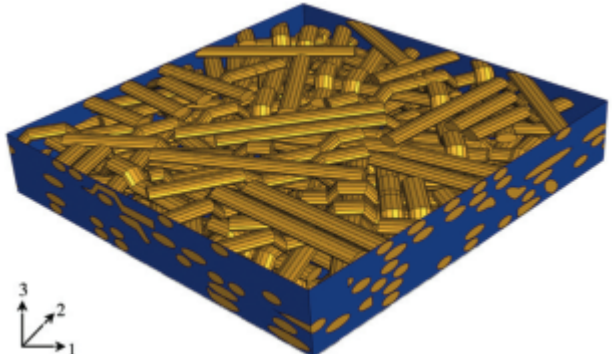


MPM3D

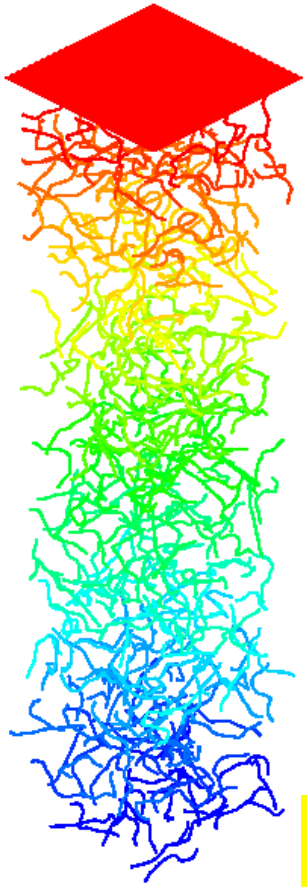
Application: Carbon Nanotube Composite

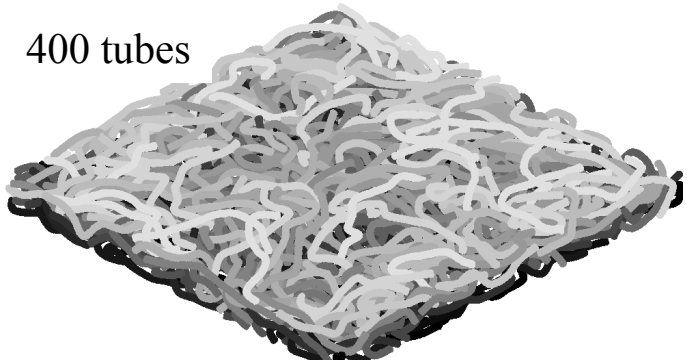
- Carbon nanotube
  - Exceptionally high stiffness, strength and resilience
  - Ideal to act as the reinforcing materials
    - 1% CNTs ➔ stiffness ↑40%, tensile strength ↑ 25%

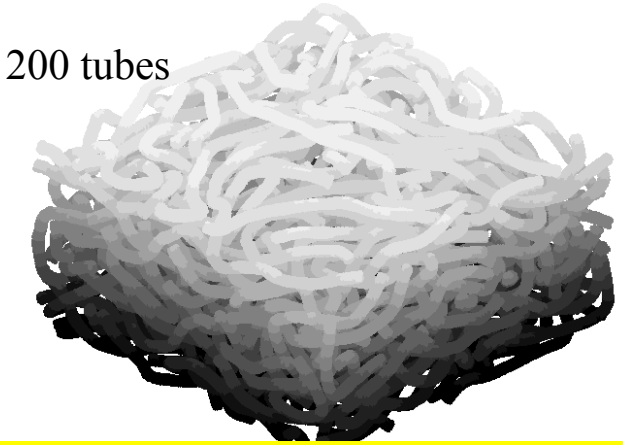




Pan, Iorga et al. ***Composites Science and Technology*** 68: 2792-2798, 2008

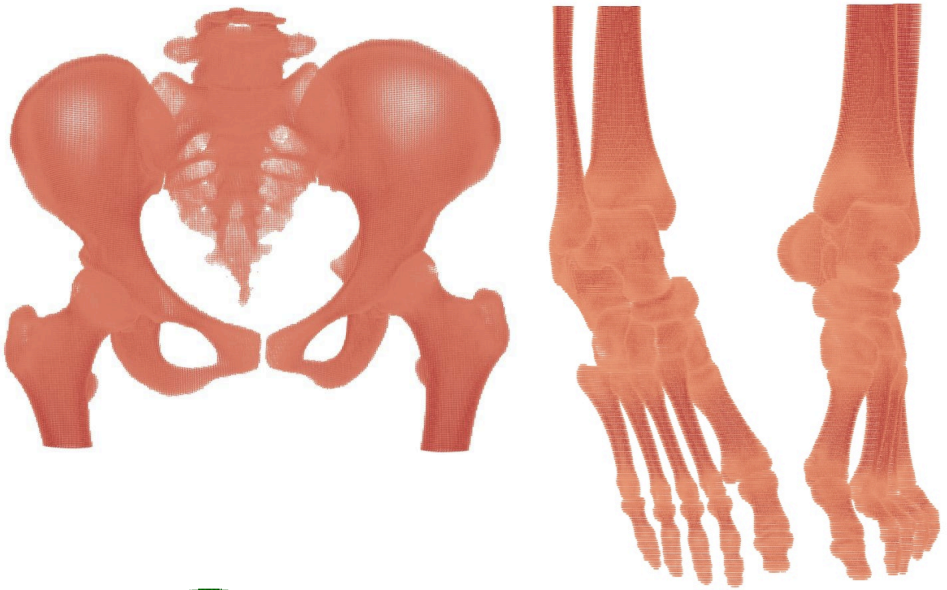


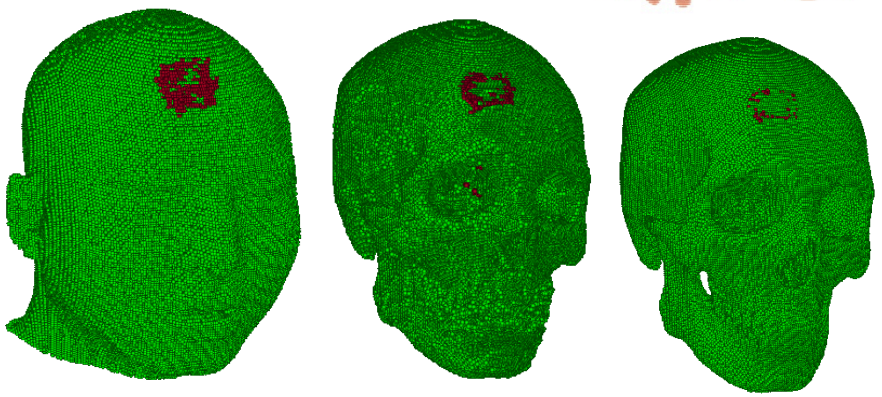




HK Wang, Y Liu, X Zhang. Numerical simulation of carbon nanotube reinforced composite by material point method. ***CMS*** 57: 23-29, 2012







Muscle Membrane Skull

SZ Zhou, X Zhang. Numerical Simulation of Human Head Impact Using the Material Point. ***International Journal of Computational Methods***. 10(4): 1350014 (2013).

MPM3D

- Compared with FEM
  - Particle quadrature instead of Gauss quadrature
  - Remesh in every time step
  - Take use of both grid and particle data
  - **Less accurate, less efficient, and more storage usage for small deformation problem**
- Combines the advantages of Lagrangian and Eulerian methods
  - History-dependent constitutive models
  - Free surfaces or multiple materials
  - No mesh distortion and element entanglement
  - **Promising in modeling extremely large deformation and material discontinuities**

Question: Could we develop a method for engineering problems with large deformation only in local region and small deformation else where ?

Answer: **MPM** + **FEM**

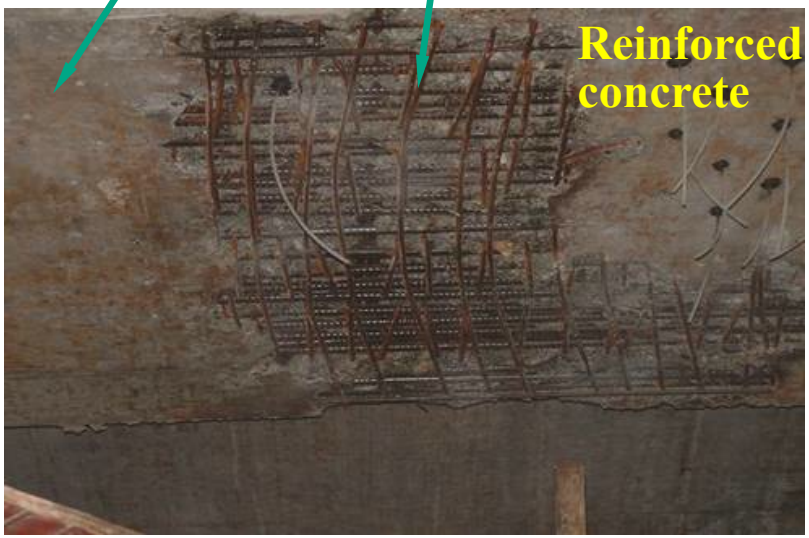
Large deformation region elsewhere

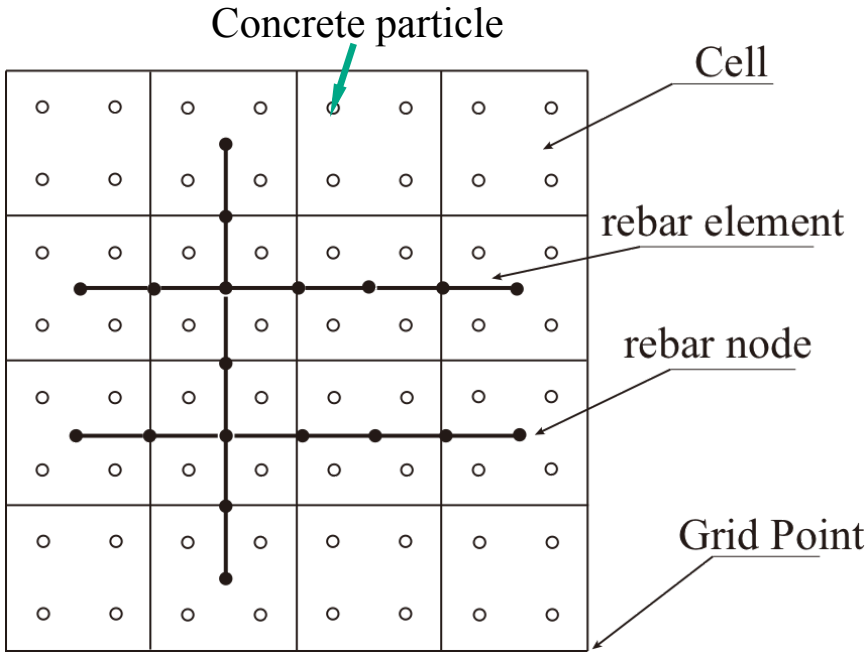
Outline

- Introduction
- MPM3D
- **FEM-MPM Coupling**
  - **Hybrid FE-MP method**
  - **Coupled FE-MP method**
  - **Material Point Finite Element Method**
  - **Adaptive FE-MP method**
- Concluding Remarks

Hybrid FE-MP method

- Plain concrete is discretized into particles
- Rebars are discretized into bar elements (rebar elements)
  - Rebar nodes carry kinematic variables
  - Rebar elements carry history variables
- All the rebar nodes and particles move in the same single-valued velocity field

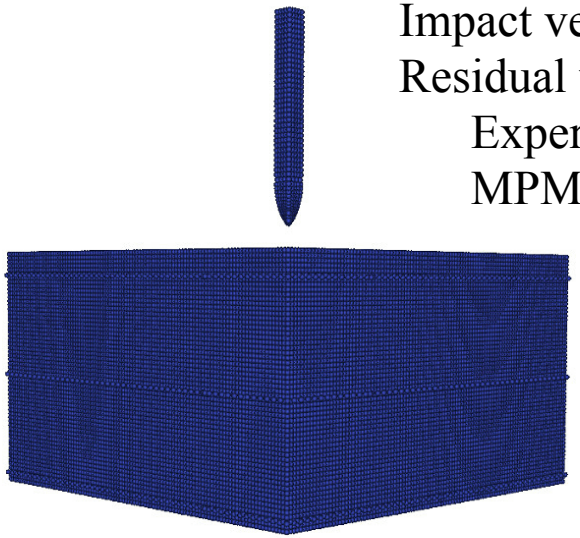




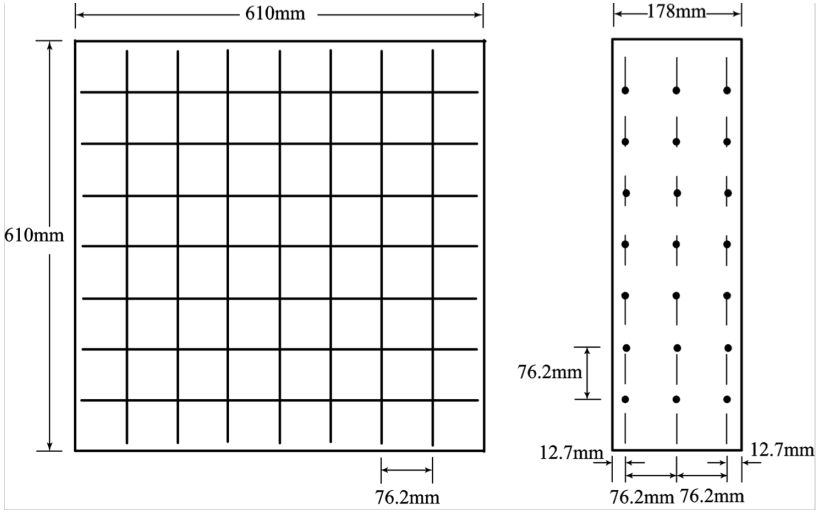
YP Lian, X Zhang et al. A FEMP method and its application in modeling dynamic response of reinforced concrete subjected to impact loading. ***CMAME***, 200(17-20): 1659-1670, 2011.

Hybrid FE-MP method

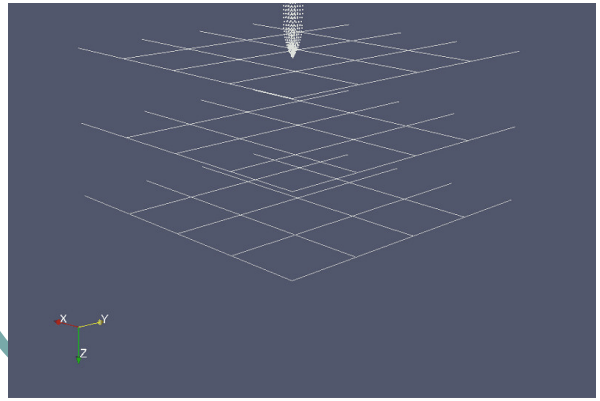
Penetration: C48 RC

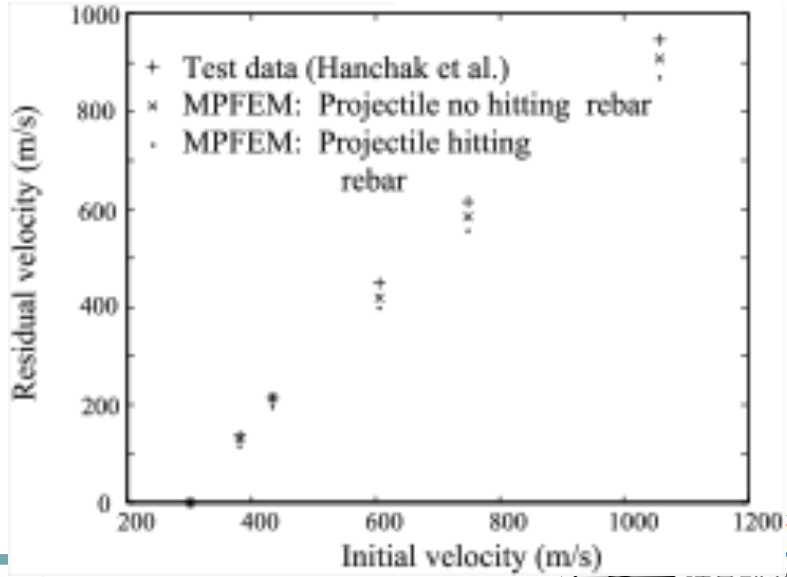


Impact velocity: 746 m/s
Residual velocity
Experiment: 615m/s
MPM3D: 585m/s



Hanchak, Forrestal et al. ***IJIE*** 1992; 12(1): 1-7

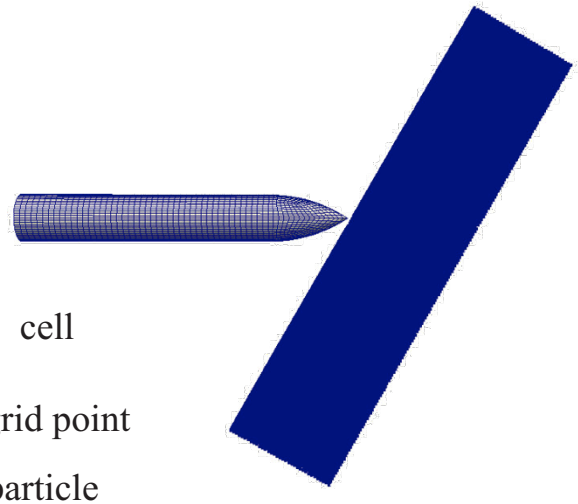


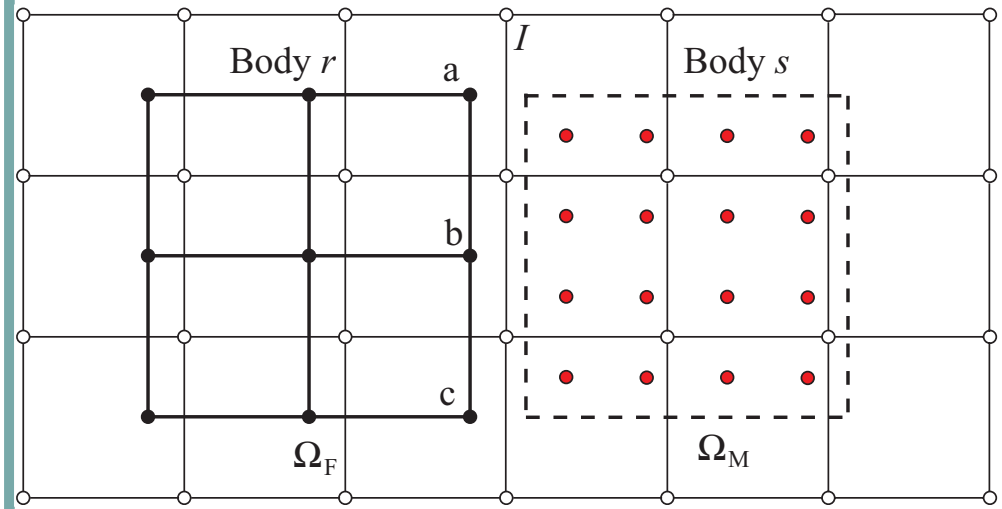


Coupled FE-MP method



YP Lian, X Zhang et al. Coupling of finite element method with material point method by local multi-mesh contact method. ***CMAME*** 200: 3482-3494, 2011





cell

grid point

particle

element

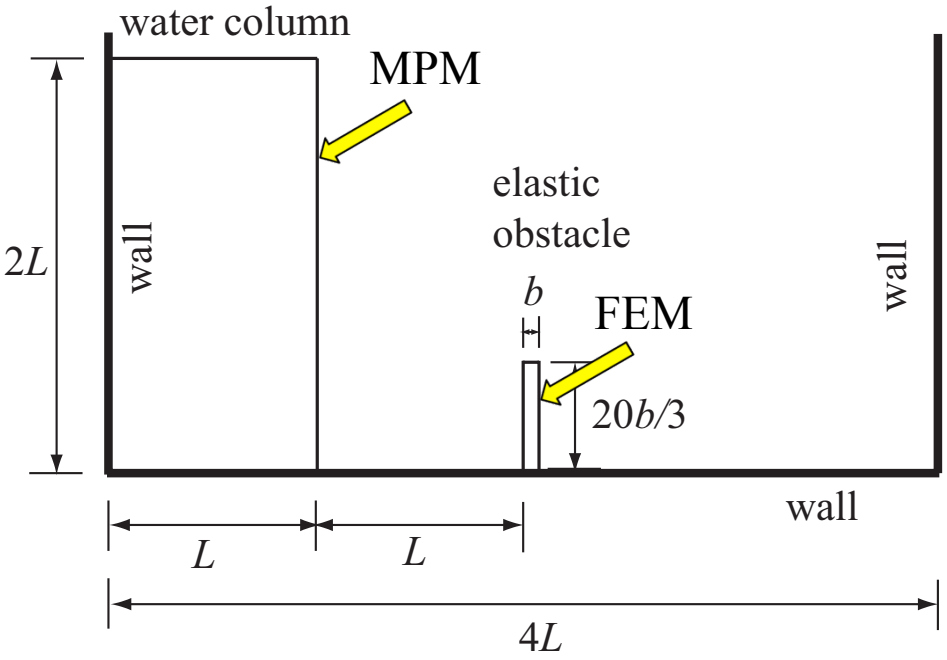
FE node

a,b,c: hybrid nodes

- Discretize different bodies by FEM and MPM, respectively
- Interaction between bodies is handled by the local multi-mesh contact method in the framework of MPM

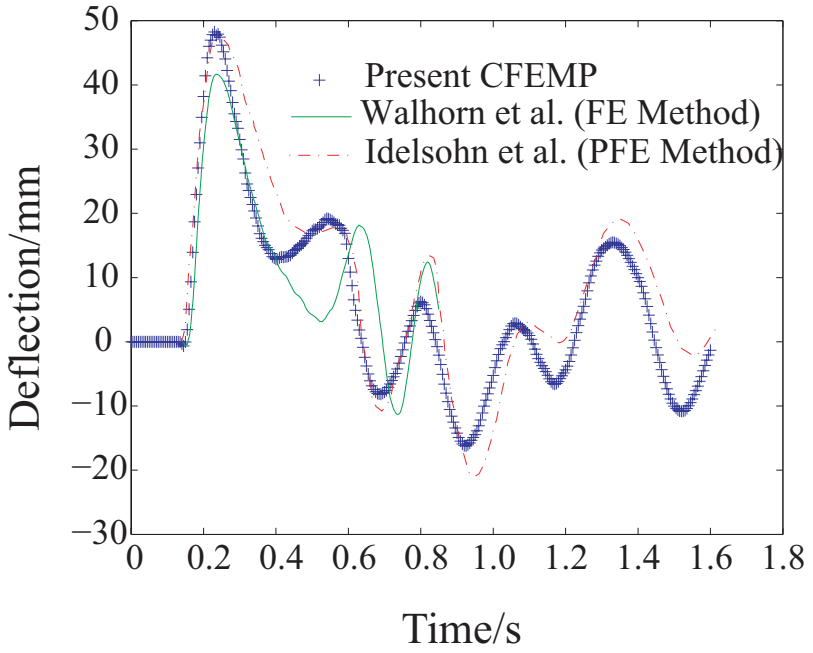
Coupled FE-MP method

Collapse of water column with a flexible obstacle

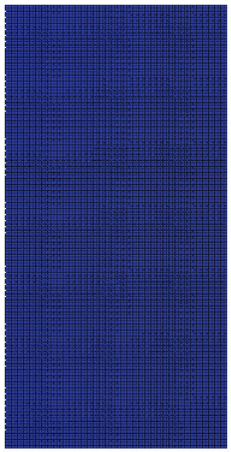


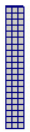
Water column:
Particles: 21,316

Flexible obstacle:
Elements: 120



Time history of the displacement of the upper left corner of the obstacle

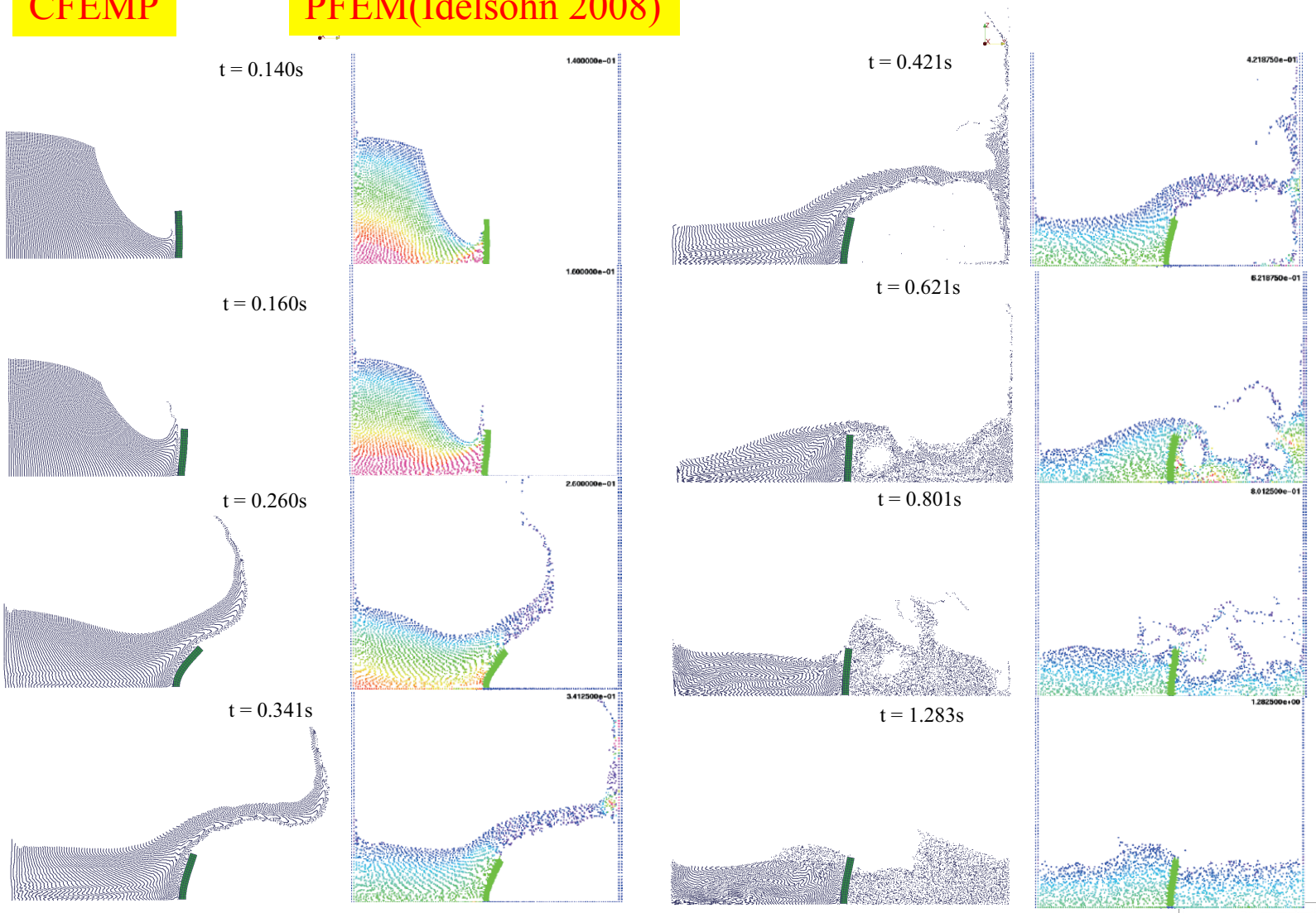




Coupled FE-MP method

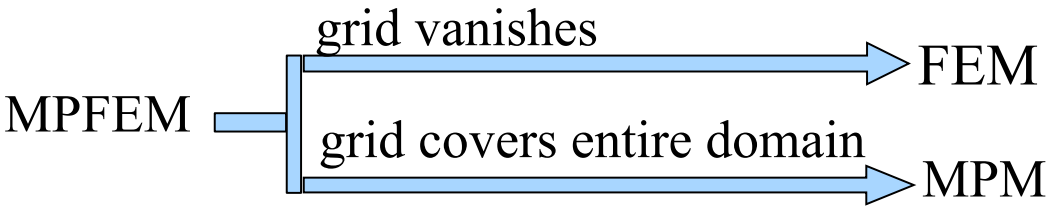
CFEMP

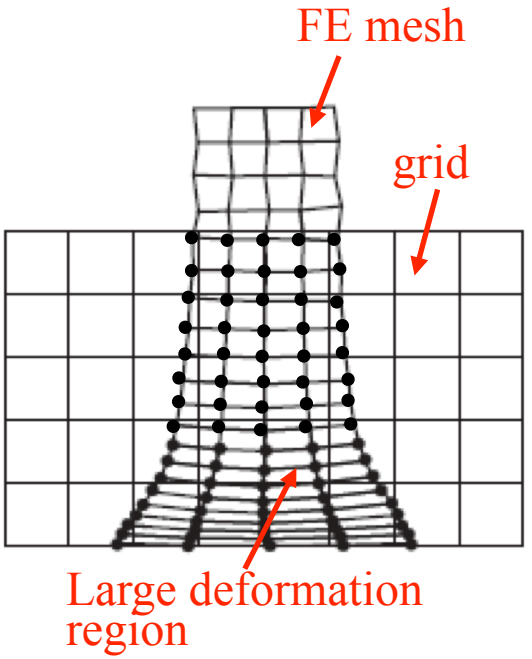
PFEM(Idelsohn 2008)

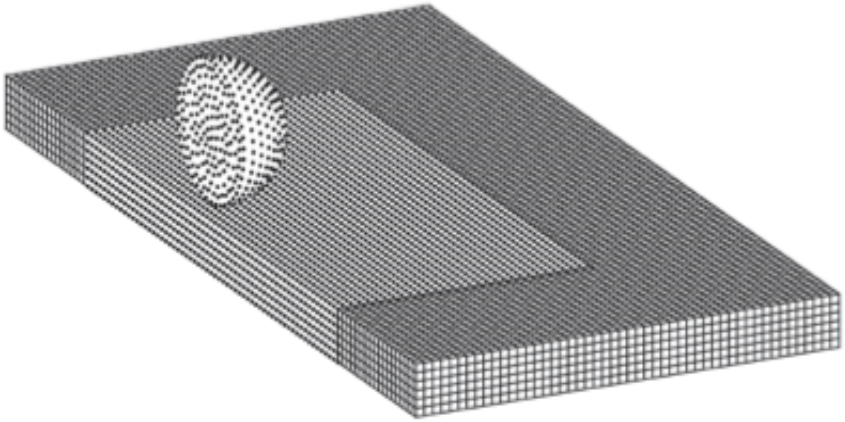


Material Point Finite Element Method

- Material domain is discretized by a mesh of finite elements
- A predefined grid is placed in the potential large deformation zone
- Momentum equations are solved on
  - Background grid — large deformation zone
  - FE mesh — elsewhere



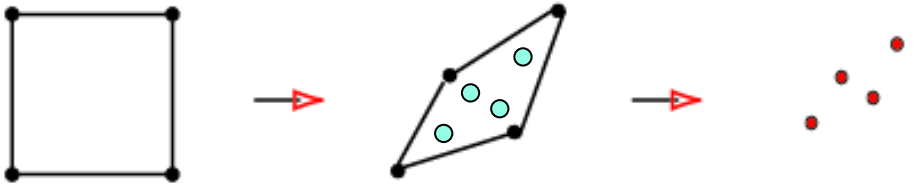


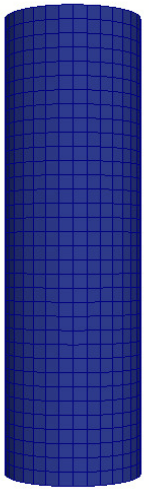


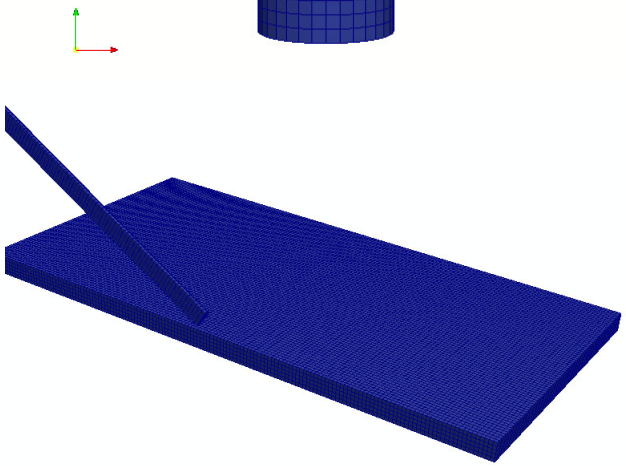
X Zhang, K Y Sze, et al. An explicit material point finite element method for hyper velocity impact, ***IJNME*** 2006; 66:689–706

Adaptive FE - MP Method

- Distorted elements are adaptively converted into MPM particles
  - When to convert – avoid element distortion
    - Effective plastic strain
    - Critical time step
  - How to convert
    - Conservation of mass/momentum/ energy
    - New particles are placed at (±0.5, ±0.5, ±0.5)







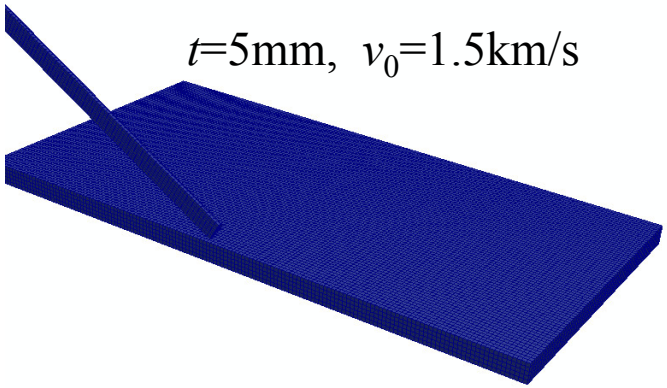
  - After conversion
    - Particles and remaining elements are coupled by the local multi-mesh contact method

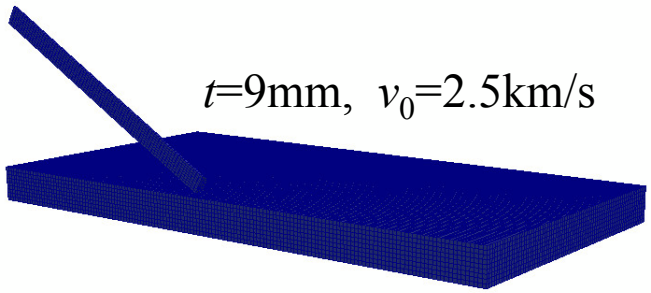
YP Lian, X Zhang et al. An adaptive finite element material point method and its application in extreme deformation problems. ***CMAME*** 241-244 (1): 275~285, 2012

Adaptive FE - MP Method

Perforation

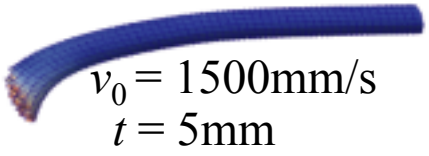
Projectile: Tungsten alloy
Target: Armor steel
Incidence angle = 60°

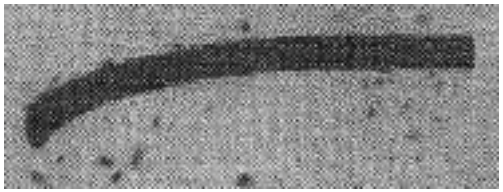


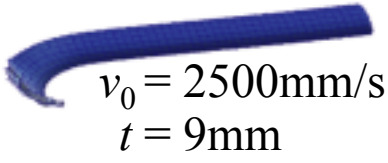


Cases		Residual length L/L_0	Residual velocity v/v_0
t=5 v_0=1.5	Experiment*	0.85	0.97
	MPM3D	0.82	0.96
t=9 v_0=2.5	Experiment*	0.76	0.99
	MPM3D	0.72	0.97

* Lars Holmberg, Patrik Lundberg et al. ***14th Inter. Sympo. Ballistics.*** 1993; 515-524





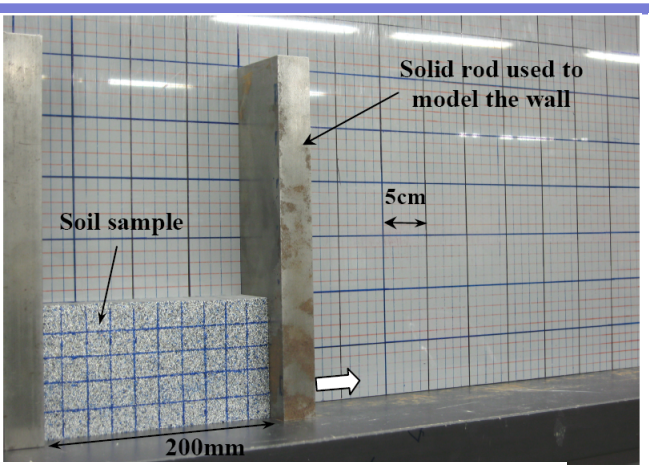




	Δt_{max}(μs)	Δt_{min}(μs)	Steps	CPU (s)
MPM	6.45e-2	3.71e-2	1180	2241
AFEMP	3.67e-2	1.52e-2	2208	838

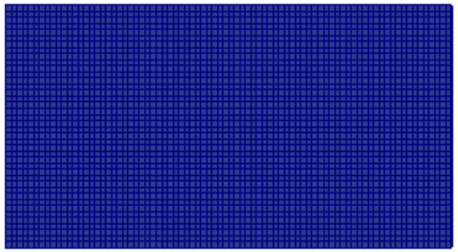
Adaptive FE - MP Method

Failure process of non-cohesive soil



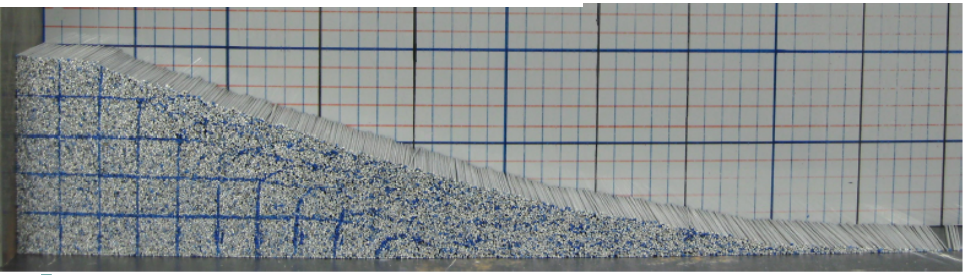
Experiments: Bui H H, Ritsumeikan University, 2006

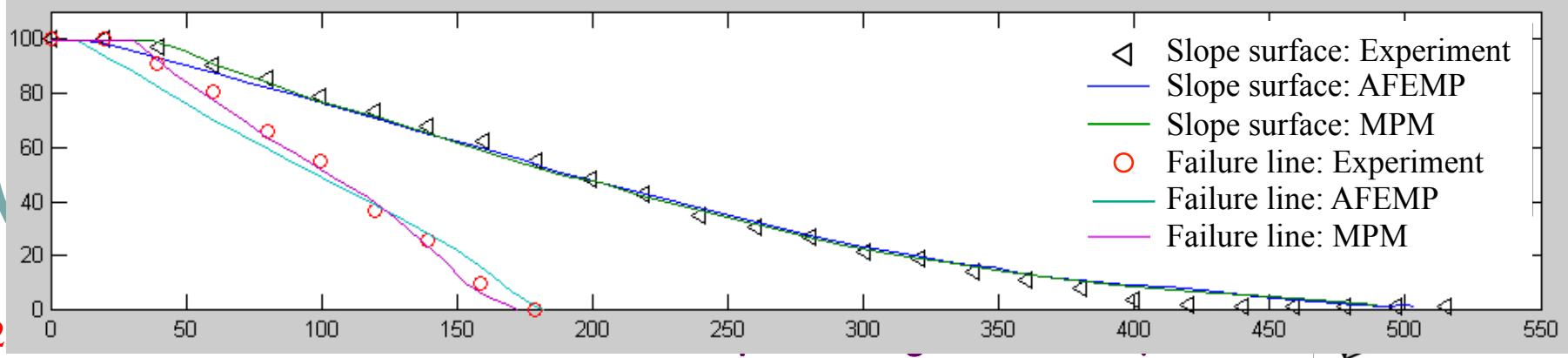
	MPM	AFEMP
Δt_{max} (μs)	48.41	48.41
Δt_{min} (μs)	29.94	22.85
Steps	34765	47402
CPU (s)	1791	1123



K=0.7MPa，ρ=2.65 g/cm^3

φ = 19.8°, c = 0kPa, ψ =0°





Outline

- Introduction
- MPM3D
- FEM-MPM Coupling
  - Hybrid FE-MP method
  - Coupled FE-MP method
  - Material Point Finite Element Method
  - Adaptive FE-MP method
- **Concluding Remarks**

Concluding Remarks

- MPM is less accurate, less efficient, and more storage usage than FEM for small deformation problem
  - Efficient implementation of MPM
  - Parallel computing
  - Combined with FEM
  - Combined with FDM
- Combines the advantages of Lagrangian and Eulerian methods
  - Impact simulation
  - Crush simulation
  - Blast simulation
- Much easier to generate MPM particles than FEM mesh
  - Carbon nanotube composite
  - Digital human modeling





Thanks for your attention!





Introduction

- In each time step

FEM

- Particles are attached rigidly to a predefined regular grid. State variables of particles are mapped to the grid nodes to establish their momentum equations

$$m_I^k = \sum_p m_p N_{Ip} \qquad f_{iI}^{\text{int}} = -\sum_p N_{Ip,j}\sigma_{ijp}\frac{m_p}{\rho_p}$$

Particle quadrature

$$f_{iI}^{\text{int}} = -\int_V N_{I,j}\sigma_{ij}dV$$

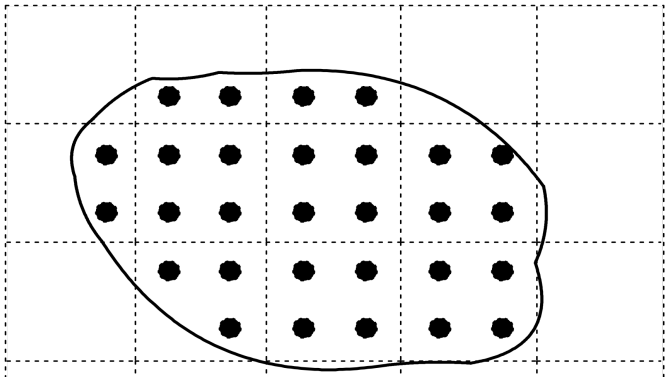
- After solving momentum equations on grid nodes, results are mapped back to the particles to update their states.

$$v_{ip}^{k+1/2} = v_{ip}^k + \sum_I \frac{f_{iI}^k}{m_I^k} N_{Ip}\Delta t$$

$$x_{ip}^{k+1} = x_{ip}^k + \sum_I \frac{p_{iI}^{k+1/2}}{m_I^k} N_{Ip}\Delta t$$

- The deformed grid is discarded, and a new regular grid is used
  - Mesh distortion is avoided

Remesh



$$N_g = \frac{1}{4}(1+\xi\xi_g)(1+\eta\eta_g)$$

Introduction

Material Point Method	Finite Element Method
$m_I^k = \sum_p m_p N_{Ip},\ p_{iI}^k = \sum_p m_p v_{ip}^k N_{Ip}$	Nodes carry mass, momentum
$f_{iI}^{k,\text{int}} = -\sum_p N_{Ip,j}\sigma_{ijp}\frac{m_p}{\rho_p}V_p$ Particle quadrature	$f_{iI}^{k,\text{int}} = -\int_\Omega N_{I,j}\sigma_{ij}\,\mathrm{d}V = -\sum_q N_{Iq,j}\sigma_{ijq}J_q w_q V_q$
$p_{iI}^{k+1/2} = p_{iI}^k + f_{iI}^k \Delta t$	$v_{iI}^{k+1/2} = v_{iI}^k + \frac{f_{iI}^k}{m_I}\Delta t \quad x_{iI}^{k+1} = x_{iI}^{\cdot\cdot} + v_{iI}^{k+1/2}\Delta t$
$v_{ip}^{k+1/2} = v_{ip}^k + \sum_I \frac{f_{iI}^k}{m_I^k} N_{Ip}\Delta t$ Remesh $x_{ip}^{k+1} = x_{ip}^k + \sum_I \frac{p_{iI}^{k+1/2}}{m_I^k} N_{Ip}\Delta t$	Gauss points keep fixed in a element, so it is unnecessary to recalculate their coordinates

